THE SUM OF THE ANGLES OF A POLYGON

WORK SCHEME: The angles of the polygon can be a)interior or b) exterior. We can consider a polygon in the divisions of triangle, quadrilateral, or polygon with more than 4 sides, in regard to interior angles or exterior angles.

First we must establish which are the exterior and which the interior angles. When we have thus identified them, we pass to the presentation.

According to the interior angles, there is one presentation for the triangles, two for the quadrilaterals and three for the polygons. In the case of more than one presentation, the subsequent presentations represent a more advanced concept and thus a higher level. There is only one presentation for each of the divisions according to the exterior angles.

Material

1. The plane insets.
2. The box of sticks, the wooden plane.
3. The Montessori protractor.
4. Colored straws.
5. The third box of the first series of constructive triangles.
6. Three large envelopes entitled:
   1) the Sum of the Interior Angles of the Triangle
   2) the Sum of the Interior Angles of the Convex Quadrilateral
   3) the Sum of the Interior Angles of the Polygons with more than 4 Sides.

Each envelope contains a wide assortment of the named figure. In the triangle envelope we find examples of all the seven triangles of reality in at least three different sizes. (a total of at least 21) In the quadrilateral envelope we find many examples in different sizes of each of the six quadrilaterals plus the two additional trapezoids---all in at least three different sizes and with a variety of dimensions so that there are at least 24 quadrilaterals. In the polygon envelope, we find regular and irregular polygons in all different sizes representing all of the polygons from the pentagon through the decagon. The figures are cut from sturdy construction paper. The interior angles on almost all of the figures are colored in. (NOTE: in constructing these colored angles, it is important to use the same radius for all the angles of the same figure or the ensuing proofs will be off) A few of the figures in each envelope are blank so that the child may color in the angles himself.

NOTE: For the presentation of this material, certain figures from each envelope and used and cut. When the children work with the exercises later, they may make models on their own paper from those figures in the envelope; an obvious necessity in order to keep the materials available to all and to prevent the need to constantly make more. The child, in making his model, simply marks the vertices of the figure and then draws the lines with the ruler.

NOTE: Angles of one figure are always same color; indicated with colored sector.

Presentation: Sum of the Interior Angles of the Triangle

1. Using the plane insets of the geometry cabinet, review the seven triangles of reality, pointing out the interior angles.

1. What words can we use to classify this triangle? It is a right-angled scalene triangle.
These are the interior angles of the right-angled scalene triangle because they lie within the surface area of the triangl

2. Review the triangles the child has drawn in his notebook. He has indicated the size of the angles in many cases and named those angles. Now, for greater understanding, he can also identify each of the interior angles.

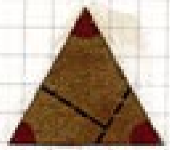
2. On each of the triangles in your notebook, you can now add this classification: interior angles.

3. Introduce the contents of the Triangle Envelope and choose one big and another size (different type triangle) triangle for analysis. Put the rest away. Classify, pointing out the interior angles.

3. This is an equilateral triangle.
This is an interior angle. . .and
this is an interior angle. . .and
this is an interior angle.
Each is colored the same.

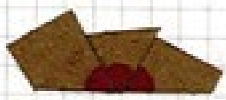
THE SUM OF THE ANGLES OF A POLYGON. . .
Presentation: Sum of the Interior Angles of the Triangle. . .

4. Cut each of the angles away.



4. We must isolate the angles (interior) for our work.
It doesn't matter if I cut close in or far away. It is only important that I cut each of the angles away.
With the three pieces, we can now see why it was necessary to color in the interior angles of our triangle.

5. Put the three interior angles together:



5. I want to know the sum of the interior angles of this triangle.
It is a straight angle---180°.

6. ACTIVITY: Using the reconstructed triangle, the child does the same experience, first making a model with the triangle as an inset, cutting his triangle out, then marking the angles and cutting them away; finally, pasting the three angles in the position as a straight angle and he writes: The sum of the interior angles of this equilateral triangle is a straight angle, that is, 180°. (He must identify the triangle specifically in his notebook because it is displayed cut up; and without such identification, it is impossible to tell which triangle it is.)

7. Repeat the experience with another of the triangles from the envelope.

After many such experiences with the various triangles, arrive at a statement with the child.

7. The sum of the interior angles of the triangle examined is still a straight angle.
In any triangle (regardless of its size) the sum of the interior angles is equal to a straight angle.

Presentation: The Sum of the Interior Angles of the Quadrilateral

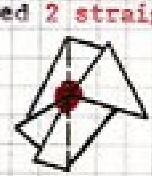
1. From the geometry cabinet, examine all the quadrilaterals, identifying the interior angles. Then do the same with those in the child's notebook.

1. I know this is a quadrilateral; that it has four equal angles, each 90°.
These are interior angles. (rectangle.)

2. Introduce the contents of the Quadrilateral envelope. Here there are many different sizes of each kind so that the generalization we make is based on wide experimental base. Choose two for analysis here and put the others away. SHOW THE FIRST LEVEL METHOD: cutting on the diagonal, then cutting away the angles of each of the triangles and finally fitting the mosaic together to achieve 2 straight angles.

2. This is a parallogram.
We want to discover the sum of the interior angles.
We can cut out quadrilateral on the diagonal and we have two triangles.
Now it's easy---we know about the interior angles of the triangle.

We can go farther, isolating the three interior angles of each triangle.
Then we put the mosaic together and we have achieved 2 straight angles.



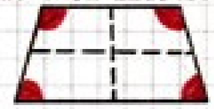
3. The child makes a model from the figure, cuts it, pastes the angles together and writes: The sum of the angles of this quadrilateral is 2 straight angles.

THE SUM OF THE ANGLES OF A POLYGON. . .
The Sum of the Interior Angles of a Quadrilateral. . .

4. SECOND LEVEL: SECOND METHOD. . . when the child has worked with and mastered the method first presented. Here we isolate the four angles of the quadrilateral and then put them together, showing a whole angle.

The child draws the figure, cuts it, pastes the angles together and writes: The sum of the interior angles of this quadrilateral is 2 straight angles.

4. This time, in order to discover the sum of the interior angles of the quadrilateral, let's isolate all the angles just as they are. . .first dividing the figure into two pieces and then into four.



Putting the angles together gives me a whole angle. I know that the whole angle is made of two straight angles.

5. We can make a proof of the two straight angles from these four angles. First showing the angles in pairs of two, then cutting the piece from the concave angle which will complete the second straight angle. Thus we equalize the two and show the two straight angles.

6. When the child has done this experience many times with different quadrilaterals he may declare that: In every quadrilateral, the sum of the interior angles is always equal to two straight angles.

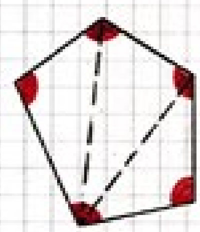
Presentation: The Sum of the Interior Angles of the Polygons with more than 4 Sides.

1. Examine the plane inset figures from the cabinet, identifying the interior angles.

2. Introduce the contents of the envelope of the polygons. NOTE: Some of these polygons have internal red circles. Choose two irregular polygons (neither equilateral nor equiangular). Put the rest aside.

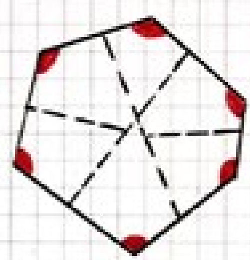
2. Here we know there are regular and irregular polygons.

3. FIRST METHOD: FIRST LEVEL.
Here we choose one vertex and draw all possible diagonals from it. We cut on each diagonal, showing the triangles and conclude that the sum of the angles (interior) of the polygon is as many straight angles as we have triangles.

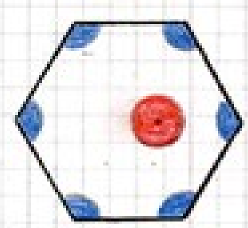


The child draws the figure, cuts, and writes: The sum of the interior angles of this pentagon is three straight angles.

4. SECOND METHOD: SECOND LEVEL.
Now we isolate each of the angles. To put all those angles together in order to show straight angles, it is necessary to cut one or two of the pieces. in order
The child writes: The sum of the interior angles of this irregular hexagon is 4 straight angles.

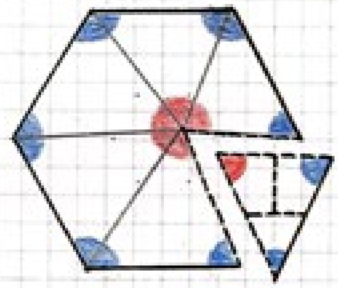


5. THIRD METHOD: THIRD LEVEL.
   a) Here we use a regular polygon for the presentation in order to have a situation later in which we have even straight angles. Later the child can do the work with the regular and irregular polygons. ALSO WE CHOOSE HERE A BLANK POLYGON, without the angles colored. SO. . .
   b) First the child draws the colored segments to show each of the interior angles.
   c) Then the child chooses a point and marks it with a red pencil ANYWHERE on the internal part of the polygon.



THE SUM OF THE ANGLES OF A POLYGON. . .
The Sum of the Interior Angles of the Polygons with more than 4 Sides. . .
THIRD METHOD: THIRD LEVEL. . .

d) Around that point which the child marks on the internal part, he draws a circle, using the same radius used to mark the angles of the polygon; and he colors the circle.



e) Unite the center of the red circle to each of the vertices of the polygon. Now we have divided the polygon into triangles.

f) Cut along the lines, cutting as pie pieces each triangle.
NOTE: here we will have six different triangles because the point was not made at the center of the polygon; in that case, the triangles will be equal.

g) Isolate each of the 3 angles of each of the triangles, cutting them away.

h) The child then separates the red angles from the others.

i) With the blue angles he makes as many straight angles as possible: 4.

j) With the red angles he can make 2---always.

k) Finally, the child draws the figure from the model used in the polygon set or another similar one from the set; he cuts it up as in the experience here, pastes the straight angles resulting together in his notebook, and writes: The sum of the interior angles of this regular hexagon is equal to 4 straight angles.

6. The child continues this work with the polygons, using the third method, always emphasizing the drawing of the internal point and constructing both the straight angles formed by the interior angles and those created by the center point. He finishes each experiment with a statement such as:
The sum of the interior angles of this decagon is eight straight angles.

7. The child works now with the form "The Sum of the Interior Angles of a Polygon" answering questions 4) How many STRAIGHT ANGLES does your polygon contain?, 5) How did you obtain the NUMBER of straight angles?, and 6) How many degrees make up the SUM of the interior angles of your polygon? He deals with these questions as regards the polygons from the triangle through the decagon, showing three proofs for each (three figures explored because the form provides for 3). He is moving towards a generalization about the interior angles of the polygon. He sees in his number of straight angles that there are always two less than there are sides of the polygon; therefore, he is able to say for n number of sides:
In the polygon with n sides, (n-2) represents the number of straight angles contained in the sum of the interior angles.

8. We help him discover how we reach this conclusion.

Sides		Str. Angles
3	(3 - 2)	1
4	(4 - 2)	2
5	(5 - 2)	3
6	(6 - 2)	4

SO the number of straight angles in the sum of the interior angles of the polygon is the number of sides minus a constant which is 2.

8. How did you obtain the number of straight angles in the polygon?
How many in the triangle? 1
the quadrilateral? 2
the pentagon? 3

9. Observe with the child one of the polygons cut as in the third method and arranged according to the straight angles. Note the two red straight angles.

9. Now we want to discover the purpose of the center point in this polygon.
We know that the number of the straight angles of this polygon is the number of its sides - 2.
The 2 we subtracted are these two straight angles formed with the red angles.
If we explore the polygons from the smallest to the largest, this constant 2 is always present.

10. A FINAL GENERALIZATION: The sum of the interior angles of every convex polygon is given by the following relation: as many straight angles as there are sides minus 2.

AN IMPORTANT FORMULA: $180^\circ \times (n - 2)$ = sum of the interior angles of the convex polygon

AGE: FIRST METHODS in work with the triangles, quadrilaterals and polygons: $8\frac{1}{2}$
SECOND ANDTHIRD LEVELS: $9\frac{1}{2}$

THE SUM OF THE EXTERIOR ANGLES OF A POLYGON

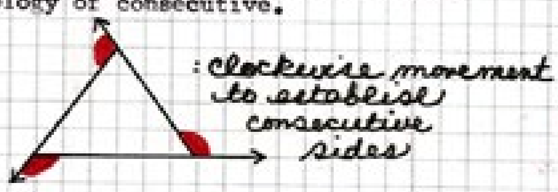
Presentation: Identifying the exterior angles

Materials
1. The box of sticks.
2. Colored straws.

NOTE: Here we are giving the concept of exterior angles.

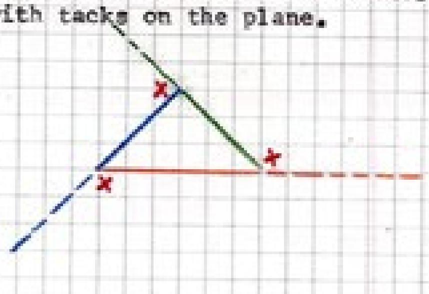
1. Choose three sticks and unite them with paper fasteners having very long legs.

2. The child identifies the sides by color and then we introduce the consecutive sides of each of those sides, noting that the direction is important and giving etymology of consecutive.



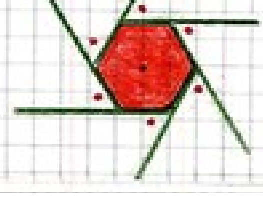
2. This is the pink side. . .the blue side. . the green side.
This is the blue side.
This is the green side.
The green side is consecutive to the blue side.
Consecutive means "to follow" or "to come after." Those words imply that we must have a point of reference.
We must establish a direction of rotation, a direction for our movement if one side is to come after another.
Now we can name each of the consecutive sides in a clockwise direction as we have done, or we may say the sequence with a counterclockwise movement.
THEN IT IS OBVIOUS THAT MR. GREEN SIDE WILL HAVE TWO CONSECUTIVE SIDES, according to the direction in which we move.

3. Choosing first one side and establishing the direction, extend first one consecutive side with a stick of equal length, then the other two. Mark the exterior angles with tacks on the plane.



3. We'll consider the green side in relation to the consecutive pink side.
We'll prolong that pink side with another stick.
An angle formed by the side of the polygon and the prolongation of its consecutive side is called an exterior angle of the polygon.
This is a side.
This is a consecutive side.
This is the prolongation of that side.
This is an exterior angle.
We have formed three exterior angles.

4. EXERCISE: Should be organized in command form. Using the plane insets from the triangle to the decagon, the child identifies the exterior angles of the polygon by forming them with straws and marking each with a tack. As a conclusion of each exploration, using the frame of the inset, the child draws each figure in his notebook, repeating the exercise. NOTE: the length of the prolongations here is not important.



THE SUM OF THE ANGLES OF THE POLYGON. . .
The Sum of the Exterior Angles of a Polygon. . .

Presentation #2: The Sum of the Exterior Angles of a Polygon

Material

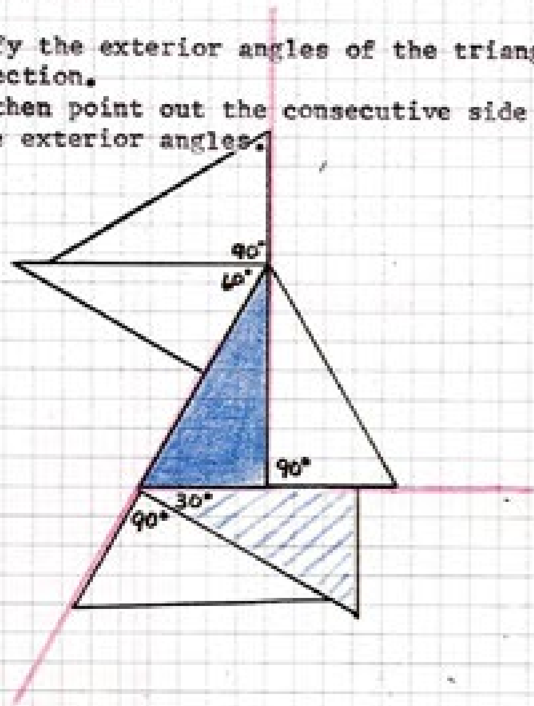
1. The box of sticks, the plane.
2. The third box of the first series of constructive triangles: the 12.
3. The Montessori protractor.

1. Take one of the triangles from the box and ask the child to identify it.

    1. It is a right-angled scalene triangle.

2. With the Montessori protractor, the child measures each of the angles and discovers interior angles of 90°, 60° and 30°.

3. Using the straws, identify the exterior angles of the triangle:
    a) Establish the direction.
    b) Choose one side, then point out the consecutive side and prolong it.
    c) Identify the three exterior angles.



4. Now, using the triangles which the child knows to be all equal---and thus to have angles of 90°, 60° and 30°; measure the size of the exterior angles. We discover:

    EXTERIOR ANGLE #1 = 90°
    EXTERIOR ANGLE #2 = 90° + 30° = 120°
    EXTERIOR ANGLE #3 = 90° + 60° = 150°

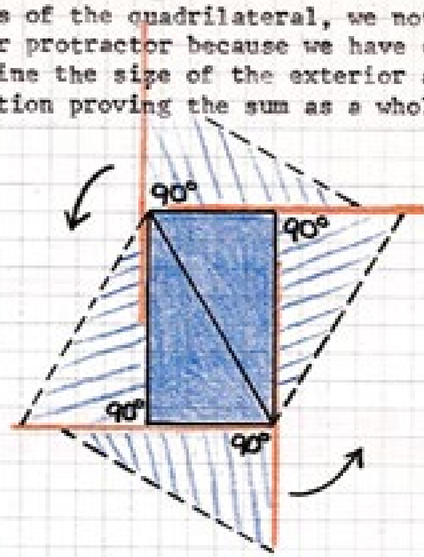
    By adding the three totals of the exterior angles, we discover that the sum is 360°.

5. We show this: TAKE THE TRIANGLE FIRST PLACED FROM THE CENTER and remove the straws. Then move the exterior angles together in the center, showing the whole angle which together they form. We conclude that the sum of the exterior angles of this triangle is a whole angle or 360°.

6. ACTIVITY: The child draws the figures in his notebook, showing the additional triangles as the sum of the exterior angles and writes: The sum of the exterior angles of the triangle is a whole angle or 360°.

Presentation #2: The Same Experience with the Quadrilaterals

As we now consider the sum of the exterior angles of the quadrilateral, we note that we are again using the constructive triangles as our protractor because we have determined the angles as 90°, 60°, and 30°. Thus we determine the size of the exterior angles. This knowledge also allows us to write a calculation proving the sum as a whole angle of 360°.



1. We begin with the simplest figure formed by these triangles: the rectangle.

2. We identify first the direction; then we name the side, the consecutive side, the prolongation of the consecutive side, the exterior angle.

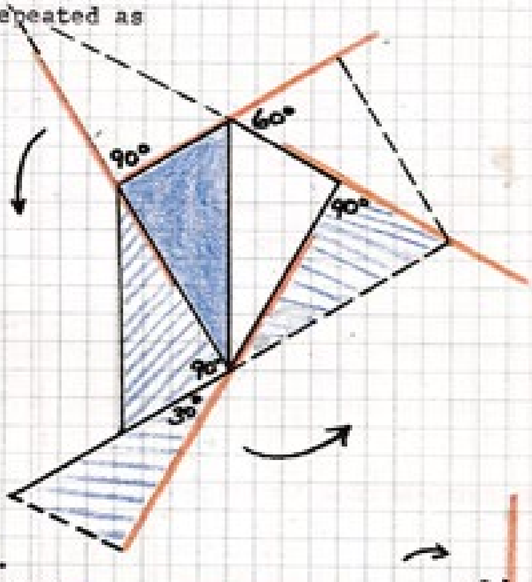
3. When the four exterior angles are identified, we show the triangles within those exterior angles, always the right angle fitting---we have four right angles.

4. We remove the center figure and put together those four right angles which form the exterior angle to obtain a whole angle. .THE SMALL STAR! .and write: $90° + 90° + 90° + 90° = 360°$

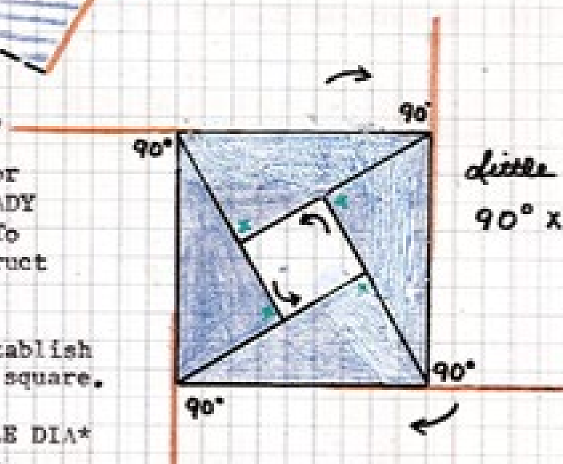
5. The child, as before, using the triangles to draw the figure formed and then to show triangles which show the exterior angles, draws the experience as above in his notebook. . .then he may draw the four exterior angles when shown together form the whole angle. OR the child may simply draw the center figure, extend the sides and identify the exterior angle. And he writes: The sum of the exterior angles of this quadrilateral is equal to 360° (a whole angle) or two straight angles.

6. THE DELTOID: the experience repeated as above for this figure.

$90° + 60° + 90° + (90° + 30°) = 360°$



7. Construct the LITTLE DIAPHRAGM. And first consider the empty square, the interior square. Identify the sides, consecutive sides and exterior angles---HERE THE DIRECTION IS ALREADY ESTABLISHED BY THE PROLONGATIONS. To form then the whole angle, we construct the LITTLE STAR.



Little Diaphram
$90° \times 4 = 360°$

8. Then form the same diaphragm and establish the exterior angles of the exterior square.

9. Repeat the experience with the MIDDLE DIAPHRAGM. Identify first the exterior angles of the internal hexagon and then the external hexagon.

10. Repeat with the BIG DIAPHRAGM.

Middle Diaphragm : $60° \times 6 = 360°$
Big Diaphragm : $30° \times 12 = 360°$