DETAILED OUTLINE FOR BIOLOGY

A. External Parts of Five Vertebrates: A Classified Nomenclature

B. First Knowledge of the Animal Kingdom
    Introduction of the pictures and labels
    Giving the definitions (stories)
    How animals satisfy their needs
        Game: identifying all the needs of one animal

C. First Classification of the Animal Kingdom: Vertebrate and Invertebrate Charts

D. The Main Characteristics of the Five Groups of Vertebrates: A Work of Comparison

E. The Chinese Box for the Animal Kingdom

F. Animal Physiology: Vital Functions

G. The Tree of Life: The Animal Kingdom
    Exercise: Compiling the classification of one species

H. The Great River: Human Physiology and Anatomy

J. The Botany Nomenclature

K. The First Knowledge of the Plant Kingdom
    Introduction of the picture cards, labels, stories
    How plants satisfy their needs

L. The First Classification of the Plant Kingdom
    General plan of the botany chart
    Proceeding to the details
    Introduction and work with the picture cards and definitions of the orders

M. Physiology of Plants and Corresponding Charts
    Functions of the Vegetative Life
        Chart #1: The needs of the plant
        Chart #2: Roots looking for water
        Chart #3: Roots overcoming obstacles
        Chart #4: Leaves act as rain spouts for thirsty roots
        Chart #5: The nitrogen cycle
        Chart #6: Osmotic pressure
        Chart #7: Plants need more water in the sun
        Chart #8: The plant seeks the light
        Chart #9: Photosynthesis
    Functions of the Life of Relation: Movement and Sensitivity
        Chart #10: Dispersion of seeds
        Chart #11: Support
        Chart #12: How roots provide support
        Chart #13: How plants defend themselves
        Chart #17: The cosmic work of roots: terracing
    Functions of the Preservation of the Species
        Chart #14: Alternation of generation in the fern
        Chart #15: Love in plants
        Chart #16: Monocotyledon/Dicotyledon

N. The Chinese Box for the Plant Kingdom

O. The Tree of Life: Plant Kingdom

P. Ecology
    The Concept of an Ecosystem
    An Ocean Ecosystem
    An Ecosystem in the Woods
    The Cosmic Idea

C

DETAILED OUTLINE FOR GEOMETRY

1. Geometry Cabinet

> Identification, etymology
> The commands

2. Constructive Triangles, First Series

> Box #1: Forming figures
> Box #2: Exploration without guide lines; characteristics
> Box #2: Variations
> Box #3: The stars and the diaphragms
> Box #3: Variations on the stars

3. CN Fundamental Concepts/Study of Lines
> Point, Line, Surface, Solid
> > Giving the concept
> > With the decimal materials
> Straight and curved line
> > With the plane insets
> > With the string
> > With the sticks
> > In the environment
> > Grammar analysis
> Positions of a Straight Line
> > Concepts of horizontal, vertical, oblique
> > On the plane
> > In the environment
> > Grammar analysis
> Parts of the straight line
> > Concepts of ray, origin, line segment, end-point
> Two straight lines on the same plane
> > Concepts of parallel, convergent, divergent

3a. CN Perpendicular and Oblique Lines

3b. CN Two Non-parallel Lines Cut by a Transversal

> Interior and exterior angles
> Interior and exterior alternate angles
> Interior/exterior angles on the same side of the transversal
> Corresponding angles
> Examining the different roles one angle can play
> Grammar analysis

3c. CN Two Parallel Straight Lines Cut by a Transversal

> Equality of alternate angles
> Equality of corresponding angles
> Straight, angle formed by pair of interior/exterior angles on the
> same side of the transversal

4. CN The Study of Angles

> Concepts of whole, straight, right, acute, obtuse
> Measuring the angles of the plane insets

4a. The Measuring of Angles/Use of the Protractor

> History of and concept of degree
> The Montessori protractor
> Measuring the insets
> Introduction and comparison with the regular protractor
> The four operations: addition, subtraction, multiplication, bisecting angle
> Other protractors

4b. CN Relationship Between Two Angles

> Two cases of adjacent angles
> Vertical angles
> Vertical angles are equal
> > Sensorial proof
> > First calculation
> Complementary and supplementary angles

4c. CN New Definition of Angles

> Convex and reflex angles
> > According to size
> > According to the prolongation of the sides
> The concave polygon
> The new definition

4d. CN The Equality of Vertical Angles: Second calculation

5. CN Formation of the Regions

> Concepts of simple closed curved regions and polygons
> Forming the polygons with the sticks

6. CN The Systematic Analysis of Triangles

> According to the sides
> According to the angles
> According to sides and angles/the 7 triangles of reality
> > The hopeless search for an eighth with the sticks
> Relationship of the lengths of the sides of jointed triangles: Sensorial
> > Scalene triangle: Cases of impossibility, limit, possibility
> > Isosceles triangle: Impossibility, limit, possibility
> The unique property of the equilateral triangle
> The specific nomenclature of the triangle
> The study of altitudes
> > Identification of interior and exterior altitudes
> > The altitudes of the 7 triangles with the plane insets
> > With the fractional insets
> > Drawing the altitudes
> > Study of the orthocenter with paper triangles
> > Researching the other points of concurrency
> Specific nomenclature of the right-angled triangle

6a. CN Written notation of the relationship of the lengths of the sides of jointed triangles
> > Scalene triangle: Cases of impossibility, limit, possibility
> > Isosceles triangle: Impossibility, limit, possibility

7. The Concepts and Names of Congruency, Similarity and Equivalence
> > Introduction of the materials: insets
> > The concept and name of congruent
> > The concept and name of similarity
> > The concept and name of equivalence
> > Equivalent figures with the insets

7a. Further Study of Similarity and Equivalence

> The similarity of rectangles, of squares, of triangles
> The equivalence of two halves
> Passages from one equivalent figure to the other

7b. Criteria for the Similarity of Triangles

> Constructing similar cardboard figures

8. CN Quadrilaterals

Construction and characteristics of the 6 quadrilaterals of reality
An exercise in sets with the 6 quadrilaterals
The diagonal
Matching labels and plane figures with the constructed quadrilaterals
The nomenclature of the quadrilaterals
    Interior and exterior altitudes
    The second diagonal
Printed form: Sum of the interior angles of the polygon (1 - 3)
Construction of the four trapezoids
    Grammar analysis

9. Constructive Triangles, Second Series

Box T:

Formation of the figures
5 new figures equivalent to the whole
Transitive property of equivalence
Relationship between lines: Between the whole and other figures
          Among the other figures

Box H_1:

Forming the figures
Equivalence according to fractional value
Relationship between lines: 2 modes
Relationship of hexagon to equilateral triangle
Inscribed triangle; circumscribed hexagon
Relationship of lines: hexagon with triangle and rhombus

Box H_2:

Formation of the figures
Fractional relationship between trapezoid and rhombus
Relationship of lines: hexagon/trapezoid, hexagon/rhombus,
          rhombus/trapezoid
Equivalence between T_2 and the trapezoid
The ratio between T_1 and T_2
    Proof #1: Trapezoid as mediator
    Proof #2: Rhombi (composed of 2/3 and 2/4) as mediators
    Proof #3: With the deltoid as mediator
Union of the three boxes
    The difference between T_1 and T_2
    The ratio between the hexagons
    The difference between H_1 and H_2
        Arithmetical calculation
        $H_1 - H_2 =$ rhombus
        Frame cut from $H_1 - H_2$
        Sensorial proof with metal insets
The Pythagorean Theorem
    Equivalence between the rhombi
    Two proofs of the ratio 3:4
        With the equilateral and obtuse-angled isosceles triangle
        With the green trapezoid
    Ratio of the equilateral triangle inscribed in another equilateral
    Ratio of the triangle inscribed in the hexagon
    Equilateral triangle built on the altitude of another equilateral
    Ratio of the two hexagons
    The inscribed square
    The Pythagorean theorem
    The Pythagorean theorem applied to other figures: sensorial

x. Mathematical notation of the Pythagorean theorem applied to other figures.

10. Regular and Irregular Polygons: Sensorial exploration

With the plane insets
Drawing the figures

10a. From the Irregular to the Regular Polygons: Second Level

Constructing the figures
Precise identification with labels
My Regular Polygons: redefinition
Nomenclature of the square and the triangle
Construction of the polygons with straws

11. Sum of Interior and Exterior Angles

Interior angles of the polygons
    Triangle, quadrilateral, polygon: First level
    Quadrilateral, polygon: Second level
    Polygon: Third level
Exterior angles of the polygon
    Identification of the exterior angles
        With the sticks
        With the plane insets
    Sum of the exterior angles of the triangle
    Sum of the exterior angles of the quadrilaterals
    Variations with the diaphragms
    Measuring the interior and exterior angles of the polygons

12. The Circle

Nomenclature of the circle and its properties
    Redefinition of the segment and the sector
Relationship between the position of the straight line and the circle: I
    External
    Tangent
    Secant
Relationship of the .. position of two circumferences: Level I
    External
    Internal
    Externally tangent
    Internally tangent
    Secant circles
    Concentric circles

12a. Relationship Between the Straight Line/Circle and Two Circles: Level II

The . . . . . . three cases of the straight line in relation to the
    circle, with radii and notation.
The six cases of the relationship of the position of two circumferences
    with radii and notation.

13. The Insets of Equivalence

Group I: Triangle

Group II: Rhombi

Group III: Common Parallelogram

Group IV: Trapezoid
    Frames 6, 7, 8 & 9
    Frame 10

Group V: Pentagon and Decagon
    Frame 11 and 12
    Frames 13, 14, 15 and 16

13. continued...

      Group VI: Theorem of the Application of Equivalence

      Sensorial Proof of Pathagorean Theorem (Frame 18)

13a. Pythagorean "Triples" (Frame 19)

   Extensions of the Pythagorean Theorem
      Using the 2nd Series of Constructive Triangles

13b. Proof of the Euclidean Theorem (Frame 20)
      Relationship of lines in equivalent quadralaterals

13c. Algebraic Proof of Pythagorean Theorem

14. Area
    The Rectangle
        How to Measure the Area of a Surface
          Inverse Rules
    The Parallelogram
    The Triangle
        Acute-angled triangle: developing three formulas
          Inverse Rules
        Right-angled triangle: three formulas
          Inverse Rules
        Obtuse-angled triangle: three formulas
    The Square
        The Formula for the Area of the Square
        Inverse Rule
    The Rhombus
        Organizing the Formulas: paper construction
        Corollary proofs with the Insets of Equivalence (Frames 2, 4, 17)
    The Trapezoid
    The Area of Polygons
        The Regular Decagon: organizing the formula with the Insets
                  of Equivalence (Frames 13, 15, 16)
        The Study of the Apothem
          Sensorial exploration of the ratio a:s
          Discovering the first digit of the constant
          Discovering the first decimal digit of the constant
        The Triangle Solution for the Area of the Polygon
    The Area of the Circle
        Sensorial Identification of the Circle as a Regular Polygon
        Measuring the Circumference: discovering
        The Formula for the Area of the Circle
          Sensorial proof
          Organizing the formula
        Area of the Parts of the Circle
          The Sector
          The Segment
          The Annulus
    The Ellipse
        Identification of the Figure: a new nomenclature
        Calculating the Area
        Constructing the Ellipse
    The Tiling Game: A Summary of the Work of Area
        Discovering those Figures which Cover a Surface
        Introduction of "Other Figures"
        Problems: Putting into relationship the calculation of the
            area of different figures

15. Volume
        Introduction: The Solid and its Transformation at the Level of
                Area and Equivalence
    The Parallelopiped
        How to Measure the Volume of a Solid
        The Formula for the Volume of the Parallelopiped
    The Right Triangular Prism
    The Right Rhombic Parallelopiped
    The Regular Right Hexagonal Prism
    Pyramids
        The Right Square Pyramid
          Sensorial experience with the hollow solids
          Introduction of the trisected cube
          The formula
        The Regular Right Triangular Pyramid
    Solids of Rotation
        The Volume of the Cylinder
        Formation of the Solids: a sensorial experience
        The Volume of the Sphere
          The spherical surface
          Introduction of the short cone
          The formula
        The Volume of the Ellipse
        The Volume of the Ovoid

16. Total and Lateral Surfaces of the Solids

17. The Study of the Five Regular Polyhedrons

18. Relationships

DETAILED OUTLINE FOR GEOGRAPHY

A. Flags, Maps and Land Forms

    Helps for the first knowledge of the earth: Globes and Geography cabinet

        Globe #1

        Globe #2: Blue/White Map of the world (drawer #1)

            Passage from the spheric surface to the plane

        Globe #3: Colored Map of the world (drawer #1)

        Globe #4

        The Geography Cabinet

            Exercises with the matching of labels: nomenclature

            Drawing activities

    The flags

        Nomenclature: relating the flag to its country

        The flag and its parts

        Flags in relation to geography

            Identifying the flags with the countries on the map

            Dolls of different countries

        Drawing the flags

        Traditions of the flags: a research

        Flags and music

    Land forms

        Six land forms of clay

            Forming the definitions

            Cutting the land forms

            Locating land forms on the globe

            Introducing the nomenclature

            Giving specific names to the land forms of the earth: maps

B. Geography Classified Nomenclature: A Third Level of the Study of Land Forms

C. The Impressionistic Charts: The Experiments II

    The Universe, The Solar System, The Earth

        Chart 1a: How small is the planet on which we live!

        Chart 2a: The sun's family

        Chart 3a: The cosmic dance

        Chart 4a: The time of the volcanoes

        Chart 5a: The beautiful daughter of the sun

        Chart 6a: What is the earth made of?

    Solar Energy and the Earth

        Chart 7a: The solar energy absorbed by the earth

        Chart 8a: Fire and ice

        Chart 11a: How the sun's rays are when they reach the earth

        Chart 12a: The sun's rays

        Chart 13a: The earth is like a mountain

        Chart 14a: The work of the solar rays in crossing the atmospher

        Chart 19a: The earth stores heat

        Chart 20a: The restoration of heat: radiation

        Chart 21a: The retained heat

        Chart 22a: Dispersed heat

    The Movements of the Earth

        Chart 9: Day and night

        Chart 10a: Which is the hottest hour of the day?

            Exercise: The time zones

        Chart 16a: The position of the earth in regard to the sun

        Chart 15a: The seasons

        Chart 17: The planisphere

        Chart 18: The distribution of the heat on earth

        Charts 1,2,3,: Summer, winter, spring and autumn

            Exercises: Constructing the seasons according to zones

        Chart 28: The Torrid Zone: the environment

        Chart 29: The Temperate Zones: the environment

        Chart 30: The Frigid Zones: the environment

C. Impressionistic Charts. . .

    The Atmosphere and its Phenomena

        Chart 24a: Winds: Why they form

        Chart 25a: Low and high pressure

        Chart 26a: Regular winds

        Chart 27a: Winds by the sea: Sea breeze

        Chart 28a: Local winds: Land breeze

        Chart 3: Winds and their direction during the equinox

        Chart 4: Winds and the distribution of precipitation

        Chart 5: Winds and the distribution of precipitation

            Exercise: Constructing the wind patterns

    The Work of the Wind

        Charts 9 and 10: Marine currents

        Chart 11: Let's destroy the rock

    The Hydrosphere and its Phenomena

        Chart 23a: How rains form

        Chart 6: Rain at sea

        Chart 7: Warm air rises: evaporation

        Chart 8: Vapor condenses

        Chart 12: Most important rivers of the world

        Charts 13, 14, 15: The earth as a sponge: great rivers of the world

        Chart 16: The work of the water

        Chart 17: The fluvial valley

        Chart 18: The canyon

        Chart 19: The mushroom: erosive phenomena

        Charts 20 and 21: Freeze and thaw

        Chart 24: The valley excavated by a glacier

        Chart 22: The glacier and the environment

        Chart 23: What the valley looks like when the glacier disappears

    The Constant Work of the Water

        Chart 25: The Cycle of Water

        Chart 26: The Game of the Water

        Chart 27: Water and plants

    RESEARCHES: corresponding to each chapter

    EXPERIMENTS: corresponding to each chapter

D. Geography Classified Nomenclature: Second Level

E. Economic Geography: WHERE do people live?

    Development of a series of maps of one's own country: To an atlas

F. Economic Geography: HOW do people live?

    The Concept of Economy

    The Concept of Imports and Exports

    The Alphabet of Economy: Analysis of What My Country Produces

        Imports and exports of my country (A map)

        Imports and exports of my country and another (A map)

        Imports and exports: of many countries (A map)

    Cosmic Considerations of Economic Geography

    The Study of Populations

    The Study of Religions

    The Study of Races

DETAILED OUTLINE FOR HISTORY

A. Exercises of Time
    The concept of time: marks on a line
    Calendar charts
    One-year time lines: the calendar
    The child's personal time line
        A short history of the child's life: a written account
        A short history in photographs
    A chart of the family
    The year and its parts: the nomenclature
    The clock

B. Preparation for Time Lines (including the work as listed in A)
    Christ: the center of man's history
    History and grammar
    The long black line
    Small experiments: Level I
    The fable: The Story of Creation: Charts 1a - 7a
    The Clock of the Eras: The corresponding strip

C. The Fundamental Needs of Man
    Introducing the concept: Exercises with the chart
    Stages in the progress of civilization: with the centuries time line
        Vertical study
        Horizontal study

D. The Time Line of Life
    General plan
    Proceeding to the details: Work with the arrows

E. The Time Line of Man: Level I
    General plan
    Proceeding to the details: First series of arrows

F. Important Concepts for the Study of Man
    The Meaning of Man's Appearance on Earth: First consideration
    Society and Civilization: defining
    The Great Civilizations: Time line

G. The Study of Civilizations: A Research Work

H. The Time Line of Man: Level II (with the second series of arrows)

J. Timeline of the Middle Ages. . . . . . .Recent History

K. The Time Line of Life: The Great Fable of Evolution

L. The Meaning of Man's Appearance on Earth: Second Presentation

M. Society and Civilization: Great Civilizations II

A. Writing
    An Introduction to the written language: Development of the alphabet
    The metal insets
    The sandpaper letters: 4 games
    The movable alphabet

B. Penmanship
    Using the small blackboards
    Capital letters
    ***Calligraphy as design

C. Spelling/Phonics
    Study of phonograms
        With the two printed alphabets and board
        With phonogram booklets and picture cards
        ***A systematic study of English phonograms

D. Composition: Indirect Preparation for Content
    Conversation: a result of experiences, research, observation
    Retelling stories
    Descriptive dialogues
    Simple Nomenclatures
    Classified Nomenclatures

E. Composition: Direct Preparation for Content
    The red and blue printed alphabets/board
        Introducing the names of the letters
        Writing a dialogue of questions and answers
        Writing numbers with roots
        Writing compound words
        Prefixes and suffixes
        Phonograms

F. Composition: Indirect Preparation for Form
    Grammar games: How words go together
        The article with the noun: last exercise
        Rearranging the noun family: from the function of the adjective
        The detective game
        The verb commands: exercise #1: from the function of the verb
        Exercises and games with the preposition: from the function of the preposition.
        Rearranging phrases with the conjunction: from the function of the conjunction.
        The use of the comma as a substitute for the conjunction.
    Question games: precise dialogue in questions and answers
        Dialogue preparation for sentence construction
        Adding adverbial extensions to a simple sentence

G. Composition: Direct Preparation for Form
    Sentence construction
    Logical analysis

H. ***Creative Writing

J. Reading Words
    Introduction to the spoken language: The importance of reading
    Baskets of word labels
        Objects in the small environment
        Identifying fixed and movable objects
        First one-word verb commands
    ←Simple and Classified Nomenclatures: picture cards and labels
    First reading booklets: three series

J. Reading Words. . .
        Examining spelling difficulties with phonogram materials
        Using the printed alphabets
                ...and tables of suffixes
                ...and tables of prefixes
                ...and tables of word families
                Compound words

K. Reading Sentences
        First sentence commands
        First books: six series
        Reading definitions in the Classified Nomenclatures
        To Speak Correctly
                The Houses of Animals
                The Voices of Animals
                A Group of Animals
                The Right Noun with the Right Verb
        Grammar functions
        Grammar boxes
        Commands for the parts of speech
        Interpretive reading
        Reading analysis work: Levels one and two.

L. ***Reading Books: Prose and Poetry Study

M. Interpretive Reading
        Simple sentences/simple actions
        Sentences with two actions
        A sequence of sentences
        Sentences with two phrases, one subordinate
        Sentences with more than one subordinate clause
        Complex actions: some phrases to memorize
        Short scenes
        ...To Creative dramatics

N. Grammar Functions
        The function of the noun
        The function of the article
                Definite and indefinite article
                Plural article
        The function of the adjective: Introducing the noun family
                Adjective game: determining the quality
                The detective game: variations
                Logical agreement between the quality and an object: noun and
                  adjective cards
                Logical agreement between a few objects and many qualities
                Reconstruction of the environment
                Adjective commands and experiments
        The function of the verb
                An impression of the difference between matter and energy
                The verb as energy: it disappears
                        Using the first verb commands
                        Logical agreement between the verb and noun: noun and verb
                          cards
                        Logical agreement between one verb and many nouns
                Special aspects of the verb
                        Transitive and intransitive: an intuition
                        Present and past: an intuition
                        Present, past and future: an intuition
                        Action can be mental
                Verb commands
                        The teacher writes commands
                        The children write commands
                        Prepared commands

N. Grammar Functions. . .
                The Noun Grammar Box
                The Adjective Grammar Box
                The Verb Grammar Box
                The Big Red Verb Box: Introduction to verb forms as a reading exercise
                The function of the preposition
                        Game: "The blue pitcher of water." . . .both positions make sense.
                        Exercises with labels and the small environment
                        Preposition commands
                The function of the adverb
                        Logical agreement between verbs and adverbs: verb and adverb cards
                        Logical agreement between one verb and many adverbs
                The function of the pronoun
                        The pronoun which takes the place of the noun.
                        The personal pronouns
                        Pronoun commands
                The function of the conjunction
                        Introducing the comma as a substitute for the "and."
                        Conjunction commands
                The function of the interjection

O. The Real Study of Grammar
                The study of the NOUN and its origins
                        The different properties of the noun
                        Another look at the noun family
                The ARTICLE: giving the names definite and indefinite
                The different properties of the ADJECTIVE
                        Comparative adjectives
                The study of the VERB
                        Agreement of the pronoun and the verb: pronoun charts
                        The Big Red Verb Box: Second Level
                                The fundamental tenses: corresponding charts
                                The nine compound tenses: corresponding charts
                                The Verb Grammar Box for the study of the tenses
                                Printed forms: Past-Present-Future and Simple-Compound
                        The whole conjugation of the verb
                                Introducing moods
                                Regular and irregular verbs
                                Transitive and intransitive verbs: corresponding charts
                                The voices of the verb
                                The study of the passive voice
                                Impersonal verbs
                                Printed forms for the complete analysis of the verb
                        The Big Red Verb Box: Third Level
                                Formation of the tenses/Complete nomenclature
                                The forms: Interrogative, Negative, Emphatic
                                Reflexive verb conjugation
                The Grammar Boxes for the further study of the PREPOSITION, ADVERB,
                        PRONOUN, CONJUNCTION, INTERJECTION
                The commands for the parts of speech: Second Level

P. Reading Analysis and Sentence Construction: First Level
                Analyzing six cases of sentences with one action: vinyl materials
                        Question/Answer form
                Sentence construction: naming the verb: wooden materials with questions
                        Question/Answer form
                Sentence construction with an indirect object
                Sentence construction with an adverbial extension
                Sentence construction with more than one adverbial extension
                Questions that the adverbial extensions answer
                        The circle chart of adverbial questions

Q. Reading Analysis, Sentence Construction and Logical Analysis: Second Level
    Giving the names of subject, predicate, direct object:
        In the work of reading analysis
        In the work of sentence construction
        Question/Answer/Part of the sentence form
    The name of the indirect object:
        In the work of reading analysis
        In the work of sentence construction
        Question/Answer/Part of the sentence form
    The name of the adverbial extensions: Logical analysis circle chart
    Passage to logical analysis: Sentence/Analysis form
    Different kinds of predicates
        The verbal predicate
        Predicate nominative
    The concept of attributes
        Special case of the attributes: predicate nominative
        Second special case: Noun in apposition
        Complements of specification and denomination
    Kinds of sentences: The study of clauses
        Analysis of the simple sentence: compound subjects, compound predi-
          cates, elliptical subject, elliptical predicate
        The compound sentence: the independent clause
        The complex sentence: the dependent clause
        Combinations

R. ***Use of the Dictionary. The Index. The Encyclopedia. Library Skills.

S. ***Outlining. Writing skills for Research.

T. ***Effective Research

DETAILED OUTLINE FOR MATHEMATICS

A. Numeration From 1 - 10
        Red and blue number rods
        Sandpaper cyphers
        Association of rods and numbers
        First memory game
        Spindles
        Cards and counters
        Odds and evens
        Second memory game
        Snake game... Search For Ten

C. Decimal System
        Knowledge of quantities
        Knowledge of symbols
        Matching symbols and quantities
            Give symbol and get quantity
            Give quantity and get symbol

B. Numeration Beyond Ten
        Formation of 11-19 with beads
        Seguin boards for teens
        Matching seguin boards with beads
        Seguin boards 10 - 90
        100 Chain
        1000 Chain

D. Concept of Operations... Addition:Subtraction
        Carrying over
        Dynamic addition
        Rich man - poor man
        1000 - 999
        Dynamic subtraction

E. Stamp and Dot Games for Addition/Subtraction
        Introduction to quantity
        Dynamic addition
        Dynamic subtraction
        Zeroes in minuend, borrowing
        The Dot Game

F. Memorization: Addition/Subtraction... Snake & Bead Exercises
        Strip board for addition
            prepared booklets (Table 1)
            problems on loose strips
            composition of 10 (Table 2)
            composition of a number (with zero)
            double of numbers
        Table 3 and loose combinations
            eliminate $\frac{1}{2}$ Table 3
        Table 4 and loose combinations
        Table 5 and loose combinations (includes exercise for 9 year olds)
        Table 6... "bingo"
            totals with loose combinations
            totals without addends
        Stacking the totals

       7 Special cases
       Word problems

       Strip board for subtraction
              prepared booklets (Table 1)
              loose combinations
              Decomposition of 9 and other minuends... without zero
                    with zero in subtrahend
       Table 2 with loose combinations
       Table 3... "bingo"
              differences with loose combinations
              differences at random
       Stacking the differences
       7 Special Cases
       Word problems

       Snake game for addition
       Commutative property with beads
       Associative property
              one set of parentheses
              two or more sets of parentheses
       Dissociative or distributive property
       Carrying
       Addition of vertical columns

       Snake game for subtraction

G. Concept of Operation... Multiplication

       Dynamic multiplication

L. Concept of Operation... Division

       Dynamic one-digit-divisor
       Decurion division
       Centurion division

H. Stamp Game for Multiplication

M. Stamp Games for Division
       Static distributive, 1-digit divisor
       Dynamic distributive, 1-digit divisor
       Dynamic group, 1-digit divisor
       Dynamic distributive, 2-digit divisor
       Dynamic group, 2-digit divisor
       Dynamic distributive, 3-digit divisor
       Dynamic group, 3-digit divisor
       3-digit divisor
              division with zero in dividend
              division with zero in divisor
              division with zero at end of divisor

J. Memorization: Multiplication

       Multiplication board with green beads
              prepared booklets (Table 1)
              loose combinations
       Table 2 as ½ of Table 1
       Table 3 and loose combinations

       Table 4 and loose combinations
       Table 5... "bingo"
              products with loose combinations
              products at random
       Stacking the products
       7 Special cases
       Word problems

       Skip counting
       Snake game
       Significance of multiplicand
       Multiplication by 10
       All combinations to form a certain product
       Small multiplication
       Inverse product
       Squaring of numbers from 1-10
       Multiplication of a binomial by a number
       Square of the binomial (Level 1 without "powers")
       Square of the trinomial (level 1 without "powers")
       Decanomial
              vertical
              horizontal
              from "angles"
       Numerical Decanomial

S. Fractions... Level 1
       Concept of fractions
              etymology
              nomenclature
              matching labels to fractions
       Numerator and denominator
       Equivalences
              abstraction and a "rule"
       Operations of addition and subtraction with same denominator
              abstraction and a "rule"
       Operation of multiplication by a whole number
              abstraction and a "rule"
       Operation of division by a whole number
              abstraction and a "rule"

N. Memorization: Division
       Division board with green beads
              finding the 36 dividends
              prepared booklets (Table 1)
              loose combinations
       Table 2... "bingo"
               quotients with loose combinations
               quotients at random
       Stacking the quotients
       Exercise with multiplication product stamps
       Prime numbers
       7 Special cases
       Word problems

O. Division with Hierchical Materials (Test Tube Division)

    1-digit divisors
        static distributive
        dynamic distributive
        dynamic group
    2-digit divisors
        static distributive
        dynamic distributive
        dynamic group
    3-digit divisors
        dynamic group
        · zero in the divisor
            in tens
            in units
            in both

P. Frames of Hierarchy

    First Bead Frame
        Introducing the frame
        Introducing the model form
        Reading, writing, and forming numbers
        Dynamic addition... horizontal
        Dynamic addition... vertical
        Dynamic subtraction... horizontal
        Dynamic subtraction... vertical
            1000 - 1
            1000 - 999
        Multiplication
            by 10, 100, 1000 using beads without frame
            by 10, 100, 1000 using frame
            3 passages of 1-digit dynamic multiplication
    Real Hierchical Materials
        Presenting the wooden materials
        Presenting the symbols
    Second Bead Frame
        Introducing the frame
        Writing quantities on form
        3 passages on forming numbers
        2-digit multiplier
            all decompositions; no partial products
            necessary decompositions; partial products
        3-digit multiplier
            zero in multiplier
            zero in multiplier and multiplicand
            exercise with adding machine (calculator)

Q. Checkerboard and Bank Game

    Introducing the Checkerboard
    2-digit multiplier (1st passage)
    3-digit multiplier (2, 3, & 4th passage)
    Drawings with the checkerboard

    Introduction to bank game material
    2-digit multiplier
    3-digit multiplier

R. The Powers & Multiplication Exercises

    Square Chain
    Square of the binomial (level 2)
    Square of the trinomial (level 2)
    Passage from one square to the successive square
    Passage from one square to a non-successive square
    Substitution Game
    Cube Chain

U. Multiples and Divisibility

    Introduction with square chain
        with bead bars for numbers under 10
        with bead bars for numbers over 10
    Using the prepared sheets
    Research for common multiples of 2 or more numbers
        Tables A & B
        Table C
    Least Common Multiple (LCM)
    Research for Divisor of a Number
    Research for divisor of 2 numbers : Greatest Common Divisor (GCD)
    Prime Factors with Table C
    Prime factors of 3 different numbers
        obtaining the LCM
        obtaining the GCD
    Finding the LCM & GCD without the pegboard
    Finding the LCM & GCD using "sets"

    Divisibility by 2 (and rule)
    Divisibility by 4 (and rule)
    Divisibility by 5 (and rule)
    Divisibility by 25 (and rule)
    Divisibility by 9 (and rule)
    9 as the result of 10-1
    10 as the result of 9 + 1
    Proof of 9 in multiplication
    Divisibility by 11 (and rule)

    Divisibility as related to multiples.

K. Flat Golden Bead Frame

    1st Passage
    2nd Passage... partial products
    3rd passage... mental carrying-over and accumulation of partial products

T. Fractions... Level 2

    Introduction: Real, Apparant, and Improper
    Addition and Subtraction with Different Denominators
        Passages to Abstraction
            Analysis of Fractions with Graph Paper
            Finding the Lowest Common Denominator
    Multiplication of a Whole Number by a Fraction
        Passage to abstraction
    Dividing a Whole Number by a Fraction
        Passage to Abstraction
    Multiplying a Fraction by a Fraction
        Passage to Abstraction
    Dividing a Fraction by a Fraction
        Passage to Abstraction

V. Decimals

    Numeration With Decimal Numbers
        1st Presentation of Quantities (through 1000th)
        1st Presentation of Symbols
        2nd Presentation of Quantities (through millionth)
            Forming and Reading Quantities on Yellow Board
        2nd Presentation of Symbols
            Formation and Reading of Symbols
            "Candelabra"
            "Pinwheel"
        Matching Quantities to Symbols
        Matching Symbols to Quantities
            "Who has more ?"
            Progressive and regressive numeration
            Fraction and power "strip"
            Comparison of decimal numbers to whole numbers

    Operations with Decimal Numbers
        Dynamic Addition
        Dynamic Subtraction

        Multiplication
            Transforming fractions to decimal numbers
            Review of multiplying by 10, 100, 1000
            Decimal Number x Whole Number
            Whole Number x Decimal Number.
            Decimal Number x Decimal number

            Decimal Checkerboard
                Introductory exercises
                Operations
                Drawings

        Division
            Whole Number ÷ Whole Number
            Decimal Number ÷ Whole Number
            Whole Number ÷ Decimal Number
            Decimal Number ÷ Decimal Number

        Invariant Property
            With skittles
            With yellow board
            As fraction calculation
            "Considering the remainder"

X. Last Passages of the Binomial and Trinomial... Squaring

    Introductory Presentations
        Multiplication of Binomials that do not yield a Square
        Binomials Greater than 10
        Square of Binomials Greater than 10

    Passage from Real Squares to Symbolic Materials (pegboard)

    Multiplication of the Binomial with Hierarchical Materials

    Passage from Numerical to Algebraic Binomial (and guide chart)

    Formation of the Trinomial (and guide chart)
        Passage to the algebraic trinomial

Y. Cross Multiplication

    Introduction: 111 x 111

    Products of Squares

Y. continued...

    Products other than Squares

    Cross Multiplication without material

Z. Square Root

    Introduction: Formation of Squares, Square Root Terminology

    From Symbolic Square to its Side

    Square Root with Golden Bead Material

    Square Root with Pegboard and Pegs

    Note on the Study of Area

    Writing of the Square Root

W. New Math

    History of Mathematics
    Psychological preparation
            concept... the number of cubes depends on the culture
            insight into past
            insight into present
    Blue and red rods
        bases less than 10
        bases greater than 10
    Matching numbers (and letters) to rods
    Cyphers and game for selecting cyphers of a certain base
    Bingo game... cyphers for bases 2 through 16
    Fundamental Law of Every System
        Spindles
    Limits of the fundamental combinations of every system
            comparison to decimal system
            vertical construction of the limits
                examining maximum total and maximum minuend
    Limits of the 4 operations
        use of large strip board for addition and subtaction in
            bases 2 through 16.

1A. Passages to the Cube Root

    From the Power of a Number to the Power of a Sum
    From the Square to its Successive Square
    From the Square to a Non-successive Square
    From the Square to its Cube
    From a Cube to the Successive Cube
        Sensorial construction
        Written operation
    From a Cube to a Non-successive Cube
        Sensorial construction
        Written operation
    Cube of the Sum of Two Terms (Binomial)
        Sensorial construction
        Written operation
        Construction first to the sides and altitude
        Written operation

1A. continued. . .

The Cube of the Trinomial
  Sensorial construction
  Written operation
  Construction to the sides and the altitude (from binomial)
  Written operation
The Hierarchical Cube of the Binomial
Introduction of the Chart of the Cubes of Numbers 1 - 9
Raising the Binomial to the Third Power with an Algebraic Value
The Hierarchical Cube of the Trinomial
The Algebraic Cube of the Trinomial

1B. Cube Root

The Concept of the Cube Root
The Cube Root of the Binomial: with "real number" materials
The Cube Root of the Binomial with the Hierarchical Materials
Moving towards Abstraction: Binomial with the Algebraic Cube
Carrying out the Cube Root (Binomial) Abstractly

The Cube Root of the Trinomial: with the "real number" materials
The Cube Root of the Trinomial with the Hierarchical Cube
Moving Towards Abstraction: The Trinomial Cube Root with the Algebraic
         Cube
The Cube Root of the Trinomial Abstractly

Particular Cases of the Cube Root
  Preparation: The Square and the Cube of 101
  Preparation: The Square and the Cube of 110
  The Cube Root with Zero as the Second Digit (Trinomial)
  The Cube Root with Zero as the Last Digit (Trinomial)

1C. Relative Numbers

The Concept of a Negative Result in the Snake Game
The Snake with the Result of Zero
The Concept of a Negative Number
Eliminating "Like" Quantities and Symbols Before the Operation
Collecting the Terms
Written Algebraic Addition
  Formulating the rules
The Different Signs of an Algebraic Operation: A series of problems
  Addition: from the problems to the rules
  Subtraction: from the problems to the rules
  Multiplication: problems from a horizontal line: rule
  Division: sensorial solutions with skittles and beads: rule