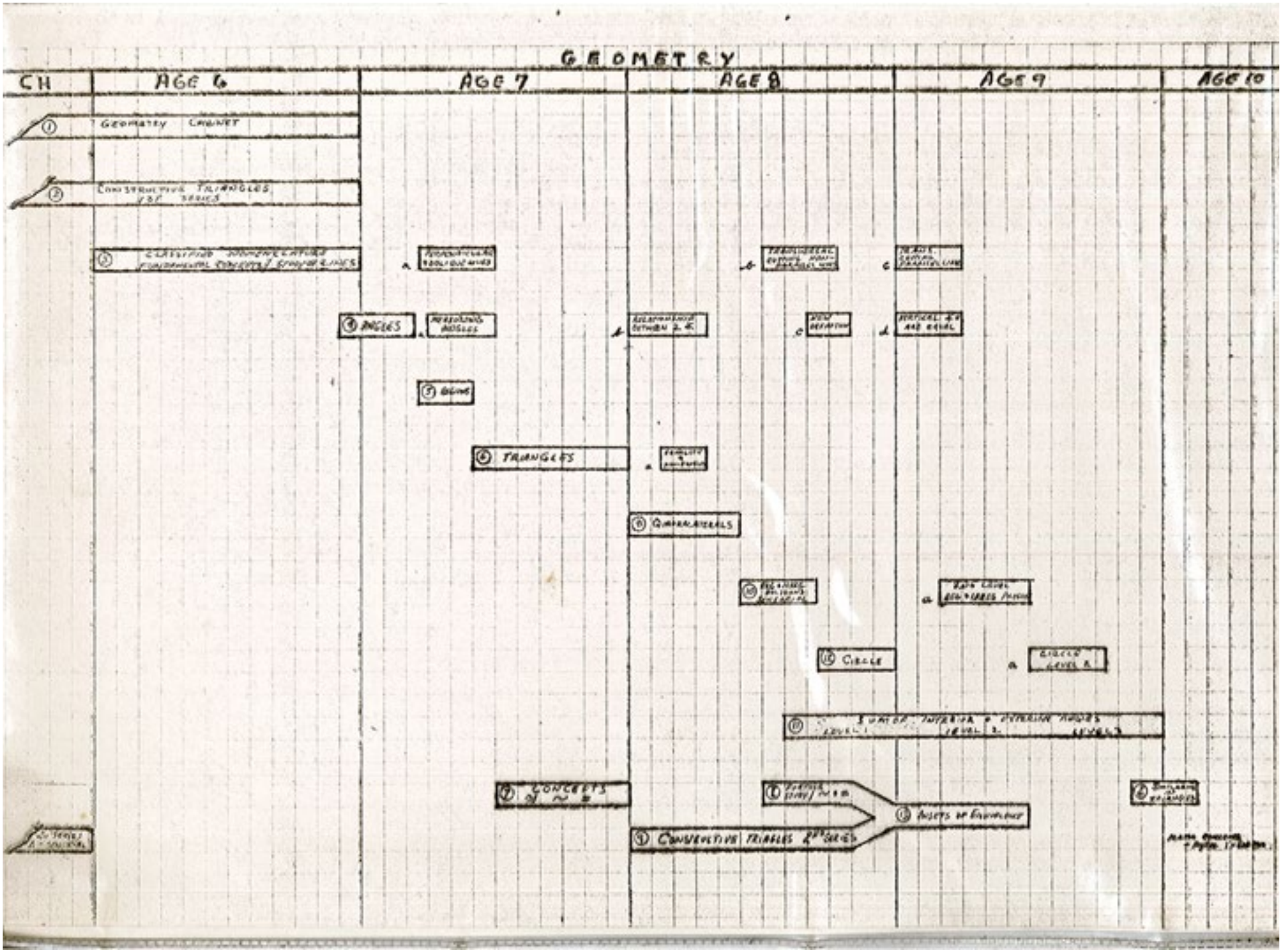
GEOMETRY
CH
AGE 6
AGE 7
AGE 8
AGE 9
AGE 10
1 GEOMETRY CABINET
2 CONSTRUCTIVE TRIANGLES 1ST SERIES
3 ANGLES
4 TRIANGLES
5 QUADRILATERALS
6 CIRCLE
7 CONCEPTS OF NO 2
8 CONSTRUCTIVE TRIANGLES 2ND SERIES

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GEOMETRY

We begin with a circle which represents the earth because geometry originally meant the measurement of the earth. As a result of man's needs, the etimology grew until it became the study of the form in reality.

Level	Age	Materials
1. The exploration of forms: to study in a creative personal way and thus to enter the study of geometry.	(Children's House) The Elementary Child	a) geometry cabinet b) constructive triangles: first series c) small solids d) plane metal insets
2. Understanding, a knowledge, of the figures in detail.	(briefly in the casa) The Elementary School	a) Classified nomenclatures of geometry
3. Concepts of congruency, similarity, and equivalence. (congruent is the modern term for equality; equal means it corresponds when superimposed)	The most important chapter for the Elementary School.	A. Dynamic aspect: a) the square and the triangle insets b) constructive triangles: second series B. Classified nomenclature applied to equivalence. a) the insets of equivalence
4. Area	Elementary	
5. Volume	Elementary	
6. Relationships	Elementary and Secondary school	

THE CONCEPTS BEHIND THIS PLAN

FIRST LEVEL: this is the exploration of forms: natural forms and those created by man. It is the study of the square and the triangle only as an experience of recognition and the sensorial. For the child, it is not important to know the characteristics of the form; but merely that the square is the square because it has this particular shape.
In this study there is an exploration of the single plane and the three-dimensional form. Giving the name and form of the three-dimensional figures.

SECOND LEVEL: The knowledge of the figures in detail is an exploration of those same figures in detail: the angles, the sides, and what are they like? Taking what the child knows and moving to to discover the limits, the composition, the special details, all the properties that man has put together on the figures.

THIRD LEVEL: The Golden Level for the Elementary School. It presents the three great concepts of geometry: congruency, similarity and equivalence. The latter of the three is the most important because, once it has been explored and developed, we can work on levels 4, 5, and 6.
There are two chapters: The dynamic aspect and the classified nomenclatures.

GEOMETRY. . .
THE PLAN. . .

The term dynamic in geometry indicates something entirely different from that which it indicates in arithmetic. It is the process in which a figure is taken apart and those pieces are reunited in a new way so that a new figure is formed; one which is equivalent to the first. So that I may say that one triangle is equivalent to a rectangle, having taken the first apart and put the pieces into a new relationship.

The second chapter, the Classified nomenclatures of geometry, combines the repetition of what is done in part A---the dynamic aspect---with the details gained during the Second Level of study; that is, the details discovered in that study of the figures. The Second Level provides the reasons why the dynamics of part A happen; and this part B, the nomenclature provides a rich synthesis of that combined information.

LEVEL FOUR AND FIVE: Because of the knowledge in Level 3, this study is possible. Level 5 is the least explored in Montessori study usually because much of her note material is sophisticated in the area of polyhedrons and the truncated pyramids and cones.

LEVEL SIX: This is the most sophisticated study; putting into relationship the generalities we have made about each figure. The figures in relation to each other produce the theorems which are here discovered.

NOTE: The constructive triangles are in two sets, each composed of various boxes:

Series 1:
1) rectangular box (colored pieces)
2) rectangular box (blue pieces)
3) rectangular box (pinwheels)

Series 2:
1) triangular box
2) small hexagonal box
3) large hexagonal box

NOTE: Contrasting the work in the Children's House and the Elementary School:

The criterion for the material used in geometry for the children's house must be modified for the elementary age. The major part of the curriculum in casa is the education of the senses: the visual sense is offered particularly rich material.

When we see an object we perceive 3 elements: dimension (how big or small), shape with its details, and color. Though generally our perception is more global, more general. Montessori arranged visual education in three levels:

for the visual sense of dimension
for the visual sense of shape
for the chromatic sense (perception of color)

We are here interested in the visual sense applied to shapes: to find within the shape the component parts. The shapes within nature (mountains, stones, flowers, leaves) and those shapes created by man and imposed on nature.

Reality is the custodian of forms great and small; tiny ones can be discovered only through the lens. The child knows the shapes of leaves and he knows the shapes created by man: the plate he eats on and the pattern on his floor. But, in his casa work, he has not studied biology or geometry, neither design nor writing. He is only doing a study of shapes and indirectly he is entering many fields. Directly he is educating his visual sense as applied to shape.

The child in the children's house is exploring and ordering shapes.
The child in the elementary school is in the process of bringing to consciousness the shapes in their details; and then in the process of exploring the relationship between figures.

DETAILED OUTLINE FOR GEOMETRY

1. Geometry Cabinet

Identification, etymology
The commands

2. Constructive Triangles, First Series

Box #1: Forming figures
Box #2: Exploration without guide lines; characteristics
Box #2: Variations
Box #3: The stars and the diaphragms
Box #3: Variations on the stars

3. CN Fundamental Concepts/Study of Lines
Point, Line, Surface, Solid
Giving the concept
With the decimal materials
Straight and curved line
With the plane insets
With the string
With the sticks
In the environment
Grammar analysis
Positions of a Straight Line
Concepts of horizontal, vertical, oblique
On the plane
In the environment
Grammar analysis
Parts of the straight line
Concepts of ray, origin, line segment, end-point
Two straight lines on the same plane
Concepts of parallel, convergent, divergent

3a. CN Perpendicular and Oblique Lines

3b. CN Two Non-parallel Lines Cut by a Transversal

Interior and exterior angles
Interior and exterior alternate angles
Interior/exterior angles on the same side of the transversal
Corresponding angles
Examining the different roles one angle can play
Grammar analysis

3c. CN Two Parallel Straight Lines Cut by a Transversal

Equality of alternate angles
Equality of corresponding angles
Straight. . . angle formed by pair of interior/exterior angles on the same side of the transversal

4. CN The Study of Angles

Concepts of whole, straight, right, acute, obtuse
Measuring the angles of the plane insets

4a. The Measuring of Angles/Use of the Protractor

History of and concept of degree
The Montessori protractor
Measuring the insets
Introduction and comparison with the regular protractor
The four operations: addition, subtraction, multiplication, bisecting angle
Other protractors

4b. CN Relationship Between Two Angles

Two cases of adjacent angles
Vertical angles
Vertical angles are equal
Sensorial proof
First calculation
Complementary and supplementary angles

4c. CN New Definition of Angles

Convex and reflex angles
According to size
According to the prolongation of the sides
The concave polygon
The new definition

4d. CN The Equality of Vertical Angles: Second calculation

5. CN Formation of the Regions

Concepts of simple closed curved regions and polygons
Forming the polygons with the sticks

6. CN The Systematic Analysis of Triangles

According to the sides
According to the angles
According to sides and angles/the 7 triangles of reality
The hopeless search for an eighth with the sticks
Relationship of the lengths of the sides of jointed triangles: Sensorial
Scalene triangle: Cases of impossibility, limit, possibility
Isosceles triangle: Impossibility, limit, possibility
The unique property of the equilateral triangle
The specific nomenclature of the triangle
The study of altitudes
Identification of interior and exterior altitudes
The altitudes of the 7 triangles with the plane insets
With the fractional insets
Drawing the altitudes
Study of the orthocenter with paper triangles
Researching the other points of concurrency
Specific nomenclature of the right-angled triangle

6a. CN Written notation of the relationship of the lengths of the sides of jointed triangles
Scalene triangle: Cases of impossibility, limit, possibility
Isosceles triangle: Impossibility, limit, possibility

7. The Concepts and Names of Congruency, Similarity and Equivalence
Introduction of the materials: insets
The concept and name of congruent
The concept and name of similarity
The concept and name of equivalence
Equivalent figures with the insets

7a. Further Study of Similarity and Equivalence

The similarity of rectangles, of squares, of triangles
The equivalence of two halves
Passages from one equivalent figure to the other

7b. Criteria for the Similarity of Triangles

Constructing similar cardboard figures

8. CN Quadrilaterals

Construction and characteristics of the 6 quadrilaterals of reality
An exercise in sets with the 6 quadrilaterals
The diagonal
Matching labels and plane figures with the constructed quadrilaterals
The nomenclature of the quadrilaterals
Interior and exterior altitudes
The second diagonal
Printed form: Sum of the interior angles of the polygon (1 - 3)
Construction of the four trapezoids
Grammar analysis

9. Constructive Triangles, Second Series

Box T:

Formation of the figures
5 new figures equivalent to the whole
Transitive property of equivalence
Relationship between lines: Between the whole and other figures
Among the other figures

Box H_1:

Forming the figures
Equivalence according to fractional value
Relationship between lines: 2 modes
Relationship of hexagon to equilateral triangle
Inscribed triangle; circumscribed hexagon
Relationship of lines: hexagon with triangle and rhombus

Box H_2:

Formation of the figures
Fractional relationship between trapezoid and rhombus
Relationship of lines: hexagon/trapezoid, hexagon/rhombus, rhombus/trapezoid
Equivalence between T_2 and the trapezoid

The ratio between T_1 and T_2
Proof #1: Trapezoid as mediator
Proof #2: Rhombi (composed of 2/3 and 2/4) as mediators
Proof #3: With the deltoid as mediator

Union of the three boxes
The difference between T_1 and T_2
The ratio between the hexagons
The difference between H_1 and H_2
Arithmetical calculation
$H_1 - H_2$ = rhombus
Frame cut from $H_1 - H_2$
Sensorial proof with metal insets

The Pythagorean Theorem
Equivalence between the rhombi
Two proofs of the ratio 3:4
With the equilateral and obtuse-angled isosceles triangl
With the green trapezoid
Ratio of the equilateral triangle inscribed in another equilatera
Ratio of the triangle inscribed in the hexagon
Equilateral triangle built on the altitude of another equilateral
Ratio of the two hexagons
The inscribed square
The Pythagorean theorem
The Pythagorean theorem applied to other figures: sensorial

x. Mathematical notation of the Pythagorean theorem applied to other figures.

10. Regular and Irregular Polygons: Sensorial exploration

With the plane insets
Drawing the figures

10a. From the Irregular to the Regular Polygons: Second Level

Constructing the figures
Precise identification with labels
My Regular Polygons: - redefinition
Nomenclature of the square and the triangle
Construction of the polygons with straws

11. Sum of Interior and Exterior Angles

Interior angles of the polygons
Triangle, quadrilateral, polygon: First level
Quadrilateral, polygon: Second level
Polygon: Third level
Exterior angles of the polygon
Identification of the exterior angles:
With the sticks
With the plane insets
Sum of the exterior angles of the triangle
Sum of the exterior angles of the quadrilateral
Variations with the diaphragm
Measuring the interior and exterior angles of the polygons

12. The Circle

Nomenclature of the circle and its properties
Redefinition of the segment and the sector
Relationship between the position of the straight line and the circles:
External
Tangent
Secant
Relationship of the position of two circumferences: Level II
External
Internal
Externally tangent
Internally tangent
Secant circles
Concentric circles

12a. Relationship Between the Straight Line/Circle and Two Circles: Level III

The . . . . . three cases of the straight line in relation to the circle, with radii and notation.
The six cases of the relationship of the position of two circumferences with radii and notation.

13. The Insets of Equivalence

Montessori Geometry

Preschool

- [I] Nomenclature: figures of plane and solid geometry
- [I] Equivalence, area, volume: sensorial

First elementary year

- [I] Nomenclature: equivalence, lines, angles

Second elementary year

- [M] Definition of plane figures
- [I] Measuring angles
- [I] 7 triangles of reality
- [I] Concepts: congruence, similarity, equivalence.

Third elementary year

- [M] Definition of Δs and quadrilaterals.
- [I] Regular and irregular polygons
- [I] Area
- [I] Definition & nomenclature: circle.
- [M] Concepts: congruence, similarity and equivalend.

[I] Introduction

[M] Mastery

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