THE STUDY OF GEOMETRY - CLASSIFIED NOMENCLATURE

DEFINITIONS

A. 1a - To the very tiny dot left by a very sharp pencil is given the name of point.
A. 1b - To the very fine trace made by a very sharp pencil is given the name of line.
A. 1c - A very thin sheet of tissue-paper, gives us the idea of surface.
A. 1d - To all things that occupy a space is given the name of solid.
B. 1a - The straight line is unlimited, direct and does not change direction throughout its length.
B. 1b - A curved line changes its direction continually from point to point.
B. 2a - A broken line is made up of line segments not going in the same direction and connected so that successive segments have an end point in common.
B. 3a - Ray is each of the two portions obtained by dividing a straight line by a point.
B. 3b - A line segment is that part of the straight line which is limited by two points.
B. 3c - The point which divides the straight line into two equal parts is given the name of origin of each ray.
B. 3d - The end points are the two points that limit a line segment.
B. 4a - A straight line is called horizontal when it follows the direction of still water. (Horizon: the line that separates the sky from the sea.)
B. 4b - A straight line is called vertical when it follows the direction of a plumb line.
B. 4c - A straight line is called oblique when it follows neither the direction of still water, nor the direction of a plumb line.
B. 5a - Two line segments are consecutive when they have only one extremity in common and do not lie on the same straight line.
B. 5b - Two line segments are adjacent when they have one extremity in common and lie on the same straight line.
B. 6a - Two straight lines are called parallel when lying on the same plane, as far as they go, they never meet, no matter how far they extend.
B. 6b - Two straight lines are called divergent when they go away from each other and therefore the distance between them increases.
B. 6c - Two straight lines are called convergent when they approach each other and therefore the distance between them decreases.
B. 6d - Two straight lines which meet are called oblique when the angles formed by them are not all equal.
B. 6e - Two straight lines are called perpendicular when crossing each other they form four right angles.
B. 7a - The perpendicular straight line drawn through the mid-point of a line segment is given the name of axis of the line segment.
C. 1a - Angle is each part of a plane limited by two rays having a common origin.
C. 2a - When the ray after wheeling a complete turn, is superimposed to the other ray, it forms an angle called: whole angle.
C. 2b - When two rays forming an angle are a prolongation of each other, they form a straight angle.
C. 2c - The angle which is half of the straight angle is called right angle.
C. 2d - When an angle measures less than a right angle, it is called acute angle.
C. 2e - When an angle measures more than a right angle, it is called an obtuse angle.
C. 2f - An angle which is greater than a straight angle, but less than a whole angle, is said to be a reflex angle.
C. 3a - Vertex of an angle is the common point from which the two rays forming an angle originates.
C. 3b - The sides of an angle are the two rays that form an angle.

C. 3c - The measurement o[illegible] is given the name: size and is expressed in degrees.

C. 4a - When an angle does not contain the prolungations of its sides, it is called: convex angle.

C. 4b - ———

C. 5a - ———

C. 5b - Angles having vertex and one side common are called: adjacent angles.

C. 5c - The opposite non-adjacent angles formed by two intersecting straight lines are called: vertical angles.

C. 6a - Two angles whose sum is equal to a right angle, and therefore equal to 90°, are called: complementary angles.

C. 6b - Two angles whose sum is equal to a straight angle, and therefore equal to 180°, are called: supplementary angles.

C. 6c - When two angles are adjacent and the two other sides are opposite rays, the sum of two angles is given the name: supplementary.

C. 7a - The angles formed on the innerside of two straight lines cut by a transversal, are called: interior angles.

C. 7b - The angles formed on the outerside of the two straight lines cut by a transversal, are called: exterior angles.

C. 7c - Interior angles lying diagonally on opposite sides of the transversal, are called: alternate interior angles.

C. 7d - Interior angles lying on the same side of the transversal are called: interior angles on the same side of the transversal.

C. 7e - Exterior angles lying diagonally on opposite sides of the transversal are called: alternate exterior angles.

C. 7f - Exterior angles lying on the same side of the transversal are called: exterior angles on the same side of the transversal.

C. 7g - Two angles, one interior the other exterior, each on one of the two straight lines, and lying on the same side of the transversal, are called: corresponding angles.

C. 8a - A ray that divides an angle into two equal parts, is called: bisector.

D. 1a - Any figure bounded by a broken straight line, is called: polygon.

D. 1b - A plane figure bounded by a closed curved line is called: a simple closed curve.

D. 2a - A polygon bounded by three line segments is called: triangle.

D. 2b - A polygon bounded by four line segments is called: quadrangle or quadrilateral.

D. 2c - All the polygons limited by more than four line segments retain the general name of polygons, but each takes its particular name, according to the number of its line segments.

D. 3a - To the plane figure similar to an egg is given the name of oval.

D. 3b - To the plane figure similar to an oval having the two minor arcs equal is given the name of ellipsis.

D. 3c - To the plane figure limited by a curved line having all the points equidistant from a fixed point is given the name of circle.

E. 1a - The triangle with all of its sides equal is called: equilateral triangle.

E. 1b - The triangle with two sides equal is called: isosceles triangle.

E. 1c - The triangle with all its sides unequal, is called: scalene triangle.

E. 2a - The triangle that has one right angle, is called: right-angled triangle.

E. 2b - The triangle that has one obtuse angle is called: obtuse-angled triangle.

E. 2c - The triangle all of whose angles are acute angles is called: acute-angled triangle.

E. 3a - The part of the plane limited by the sides of a triangle is called: surface.

E. 3b - The line segments which bound a triangle are called its: sides.

E. 3c - The side opposite to each vertex may be considered as a base.

E. 3d - The total of the sides of a triangle is called its: perimeter.

E. 3e - Each part of the plane enclosed between two consecutive sides of a triangle, is called an angle.

E. 3f - The points where two sides of a triangle meet, are called: vertices (singular: vertex).

E. 3g – A line segment from any one vertex of the triangle, drawn perpendicular to the opposite side, is called its: altitude.

E. 3h – The bisector drawn from the vertex of an angle of a triangle to the opposite side is given the name of bisector of the triangle.

E. 3i – Every line segment joining a vertex to the mid-point of the opposite side, is called: median.

E. 3j – The perpendicular straight line drawn through the mid-point of a side is given the name of axis of a side of the triangle.

E. 4a – In a right-angled triangle the side opposite the right angle is called: hypotenuse.

E. 4b – In a right-angled triangle, the sides forming the right angle are called: catheti.

E. 5a – When the catheti of a right-angled triangle are equal, it takes the name: right-angled isosceles triangle.

E. 5b – When the catheti of a right-angled triangle are unequal, it takes the name: right-angled scalene triangle.

F. 1a – A quadrilateral which has no parallel sides is given the name of common quadrilateral.

F. 1b – A quadrilateral which has only two opposite sides parallel, is called: trapezoid.

F. 1c – A quadrilateral whose opposite sides are parallel, is called: common parallelogram.

F. 1d – All the quadrilaterals whose opposite sides are parallel are called: parallelograms.

F. 1e – The parallelogram which has all right angles, is called: rectangle.

F. 1f – The parallelogram which has only equal sides is called: rhombus.

F. 1g – The parallelogram which has equal sides and right angles, is called: square.

F. 2a – The part of the plane enclosed inside the parallelogram, is called: surface.

F. 2b – The line segments which bound a parallelogram, are called its: sides.

F. 2c – Each side of a parallelogram takes the name: base.

F. 2d – The total of the sides of a parallelogram, is called its: perimeter.

F. 2e – Each part of the plane enclosed between two consecutive sides of the parallelogram, is called: angle.

F. 2f – The points where two sides of a parallelogram meet, are called its: vertices (singular: vertex).

F. 2g – The perpendicular distance between two opposite sides, is called: altitude.

F. 2h – Each line segment which joins opposite vertices of a parallelogram, is called: diagonal.

F. 3a – The part of the plane enclosed inside the rectangle, is called: surface.

F. 3b – The line segments which bound a rectangle are called: sides.

F. 3c – Each side of the rectangle takes the náme: base.

F. 3d – The total of the sides of the rectnagle is called: perimeter.

F. 3e – Each part of the plane enclosed between two consecutive sides of the rectangle, is called: angle.

F. 3f – The points where two sides of a rectangle meet, are called: vertices (singular: vertex).

F. 3g – The perpendicular distance drawn between two opposite sides is called: altitude.

F. 3h – Each line segment which joins opposite vertices of a rectangle is called: diagonal.

F. 4a – The part of the plane enclosed inside the rhombus is called: surface.

F. 4b – The line segments which bound a rhombus are called: sides.

F. 4c – Each side of the rhombus takes the name: base.

F. 4d – The total of the sides of the rhombus is called: perimeter.

F. 4e – Each part of the plane enclosed between two consecutive sides of the rhombus is called: angle.

F. 4f – The points where two sides of a rhombus meet are called: vertices (singular: vertex).

F. 4g - The perpendicular distance drawn between two opposite sides is called: altitude.
F. 4h - Each line segment which joins opposite vertices of a rhombus is called: diagonal.
F. 5a - The part of the plane enclosed inside the square is called: surface.
F. 5b - The line segments which bound a square are called: sides.
F. 5c - Each side of the square takes the name: base.
F. 5d - The total of the sides of the square is called: perimeter.
F. 5e - Each part of the plane enclosed between two consecutive sides of the square is called: angle.
F. 5f - The points where two sides of a square meet, are called: vertices (singular: vertex).
F. 5g - The perpendicular distance drawn between two opposite sides is called: altitude.
F. 5h - Each line segment which joins opposite vertices of a square is called: diagonal.
F. 6a - The part of the plane enclosed inside the trapezoid is called: surface.
F. 6b - The line segments which bound a trapezoid are called: sides.
F. 6c - The parallel sides of the trapezoid are called its: bases.
The longest is called: the major base and the other is called: minor base.
F. 6d - The two non-parallel sides of a trapezoid take the name of oblique sides.
F. 6e - The total of the sides of a trapezoid is called: perimeter.
F. 6f - Each part of the plane enclosed between two consecutive sides of a trapezoid is called: angle.
F. 6g - The points where two sides of a trapezoid meet are called: vertices (singular: vertex).
F. 6h - The perpendicular distance drawn between the parallel sides of a trapezoid is called: altitude.
F. 6i - Each line segment which joins the opposite vertices of a trapezoid is called: diagonal.
F. 7a - A trapezoid whose non-parallel sides are equal is called: isosceles trapezoid.
F. 7b - The trapezoid which has two non-parallel unequal sides is called: scalene trapezoid.
F. 7c - A trapezoid having one of its non-parallel sides perpendicular to its base is called: right-angled trapezoid.
G. 1a - To a polygon having the sides and angles unequal is given the name of irregular polygon.
G. 1b - An equiangular polygon with unequal sides is still called an irregular polygon.
G. 1c - An equilateral polygon with unequal angles is always called an irregular polygon.
G. 1d - A polygon having the sides and angles equal is finally called a regular polygon.
G. 2a - The regular polygon with three sides takes the name of equilateral triangle.
G. 2b - The regular polygon with four sides takes the name of square.
G. 2c - A polygon having five sides is called: pentagon.
G. 2d - A polygon having six sides is called: hexagon.
G. 2e - A polygon having seven sides is called: heptagon.
G. 2f - A polygon having eight sides is called: octagon.
G. 2g - A polygon having nine sides is called: nonagon.
G. 2h - A polygon having the sides is called: decagon.
G. 3a - The part of the plane enclosed inside the polygon is called: surface.
G. 3b - The line segments which bound a polygon are called its: sides.
G. 3c - Each side of a polygon takes the name: base.
G. 3d - The total of the sides of a polygon is called its: perimeter.
G. 3e - Each part of the plane enclosed between two consecutive sides of the polygon is called: angle.

G. 3f - The points where two sides of a polygon meet are called its: vertices (singular: vertex).

G. 3g - A line segment drawn from one vertex to another which is not consecutive is given the name of diagonal.

G. 3h - The point which is equidistant from all the vertices and from all the sides is given the name of center of a regular polygon.

G. 3i - The line segment drawn from the center of a polygon to one of its vertices is given the name of radius of the regular polygon.

G. 3j - The perpendicular line segment drawn from the center of the regular polygon to one of its sides is given the name of apothem.

H. 1a - The part of the plane within the outline of the circle is called: surface.

H. 1b - The fixed point within the circle from which all points of the closed curve are equidistant is called: center.

H. 1c - A line segment joining the center to any point on the circumference is called: radius (plural: radii).

H. 1d - A line segment joining any two points on the circumference is called: chord.

H. 1e - A line segment passing through the center and having as end pointst the circumference is called: diameter.

H. 1f - The closed curve whose points are equidistant from the center and which limits the circle is called: circumference.

H. 1g - A part of the circumference limited by two points is called: arc.

H. 1h - Each of the two equal parts obtained by dividing the circumference along the diameter is given the name of semicircumference.

H. 1i - Each part of a circle formed by a diameter is called: semicircle.

H. 1j - The figure formed by two radii and the intercepted arc is called: sector of a circle.

H. 1k - The figure formed by a chord and its arc is called: segment of a circle.

H. 2a - The straight line having no point in common with the circumference is called: external.

H. 2b - A straight line which meets the circumference at one point only is called: tangent.

H. 2c - A straight line which intersects the circumference at two points is called: secant.

H. 3a - Two circumferences having no point in common, one being outside the other are called: external.

H. 3b - Two circumferences having no point in common, but one being inside the other are called: internal.

H. 3c - Two circumferences having only one point in common and being external to each other are called: externally tangent.

H. 3d - Two circumferences having one point in common, but one being internal to the other are called: internally tangent.

H. 3e - Two circumferences having two points in common are called: secants.

H. 3f - Circles having the same center are called: concentric.

H. 3g - The part of the plane enclosed between two concentric circumferences is called: annulus.

0

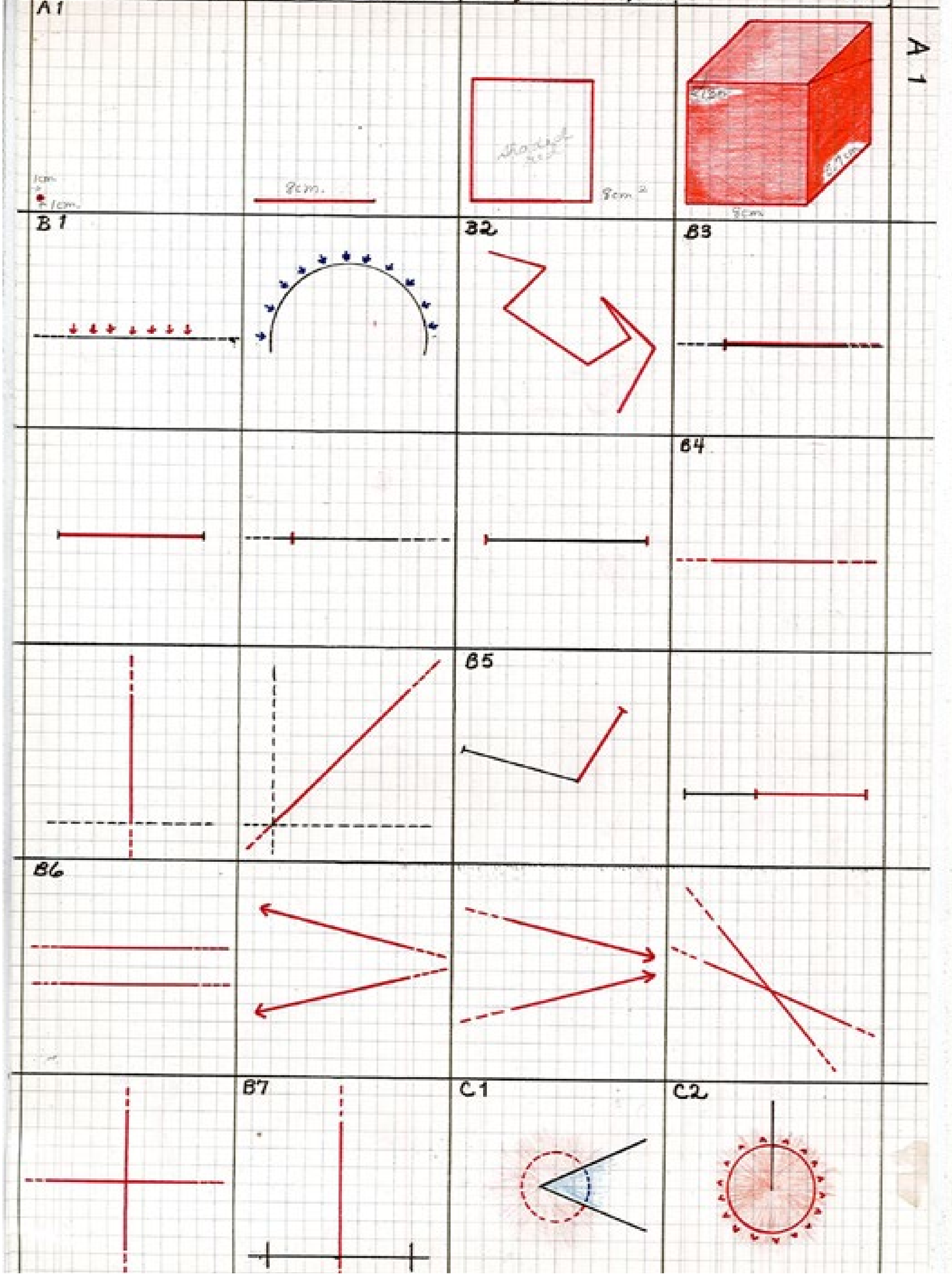
Plane Geometry
Classified Nomenclature

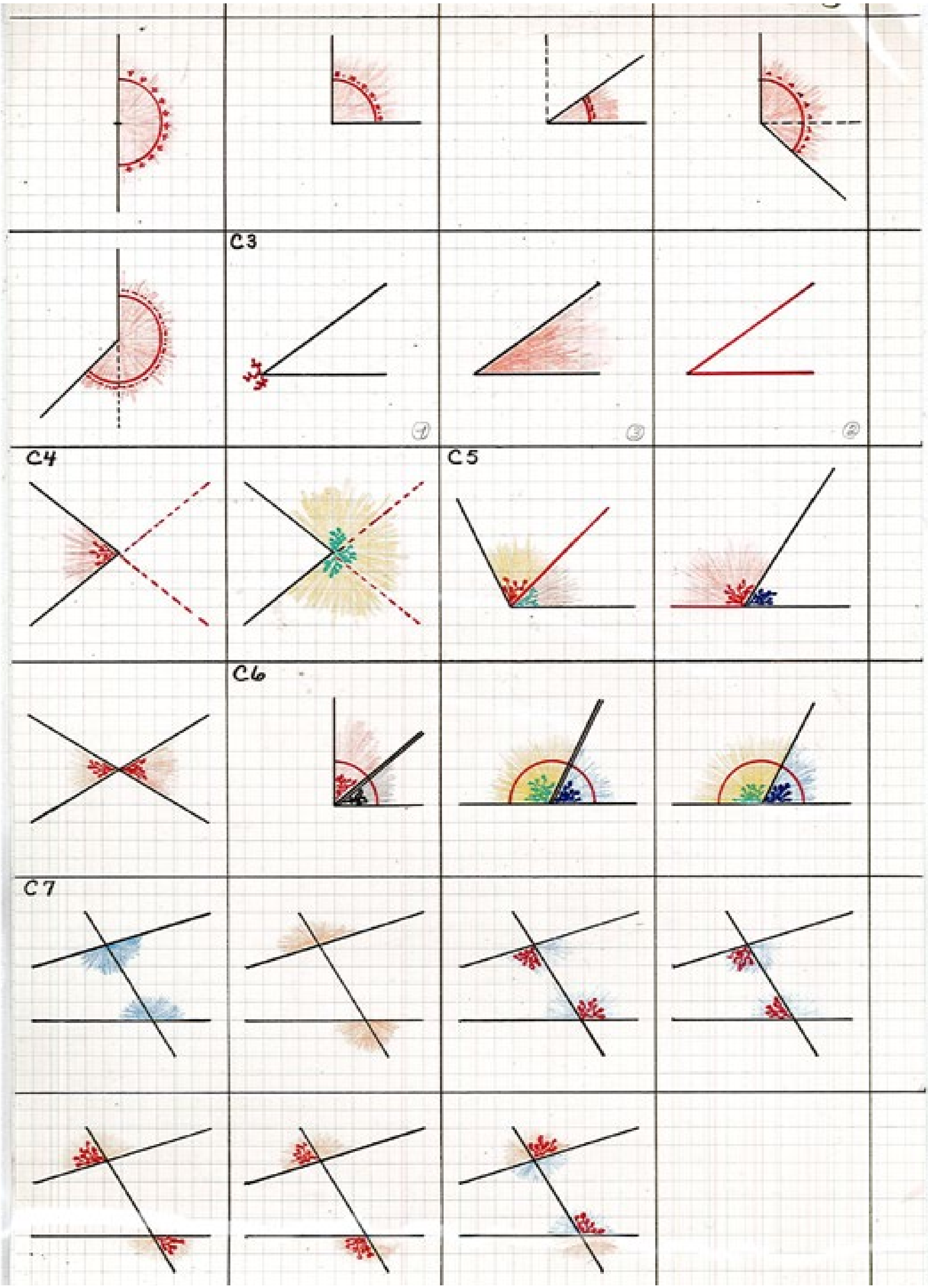
Index

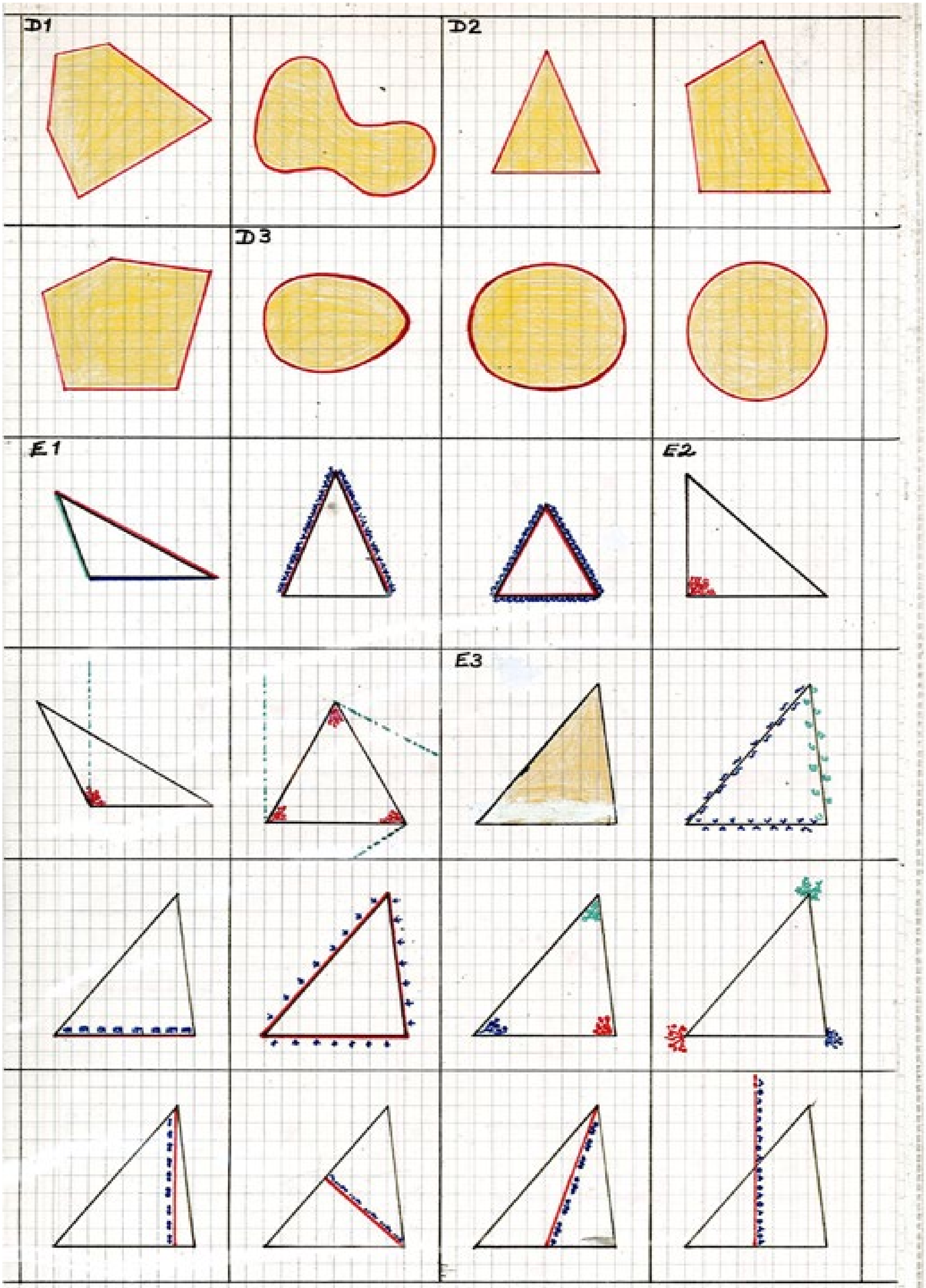
Series	Color	Subject
A 1		Basic Ideas
B 1–7		Lines
C 1–8		Angles
D 1–3		Plane Figures
E 1–5		Triangles
F 1–7		Quadrilaterals
G 1–3		Polygons
H 1–3		Circle

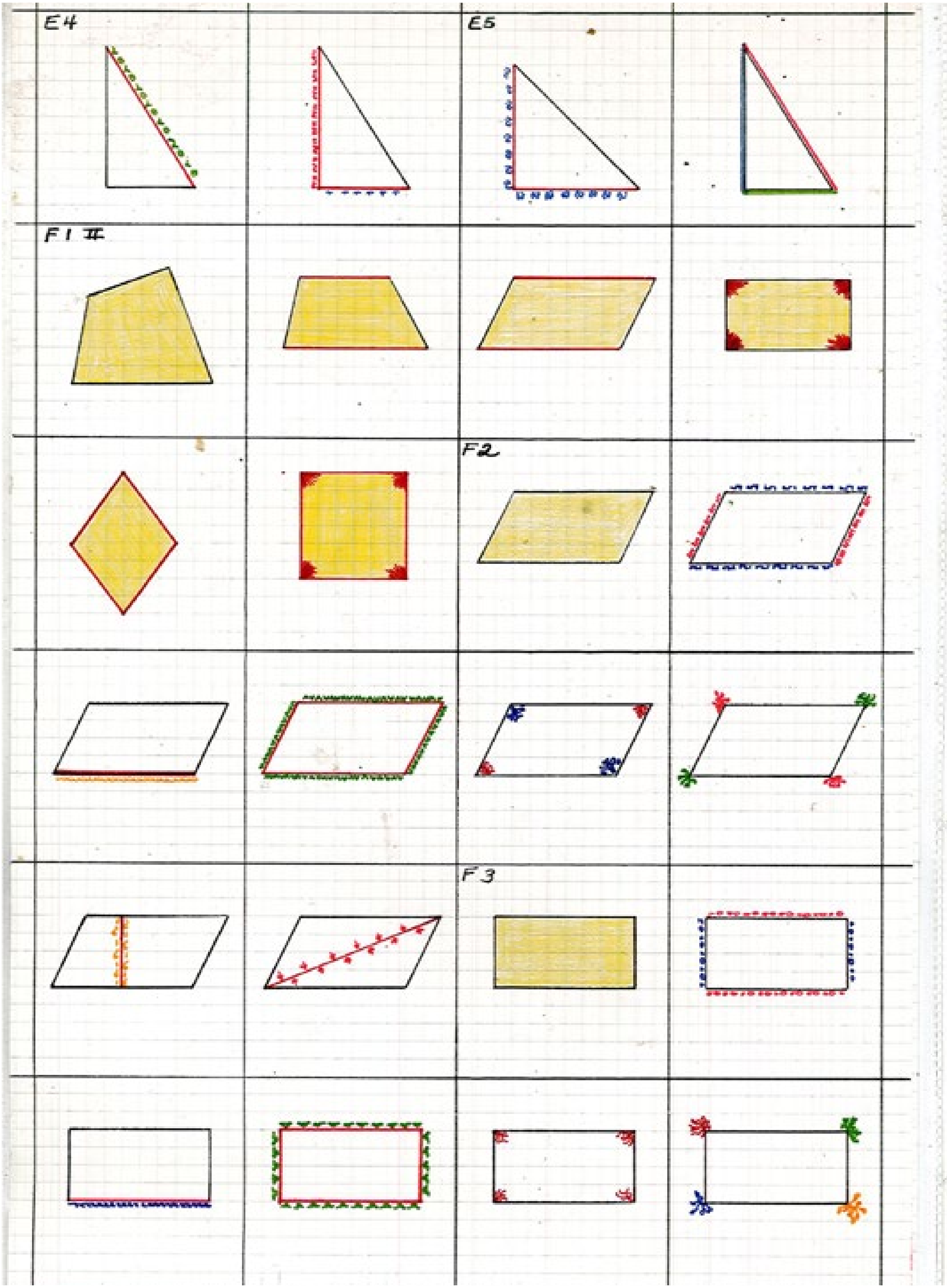
1

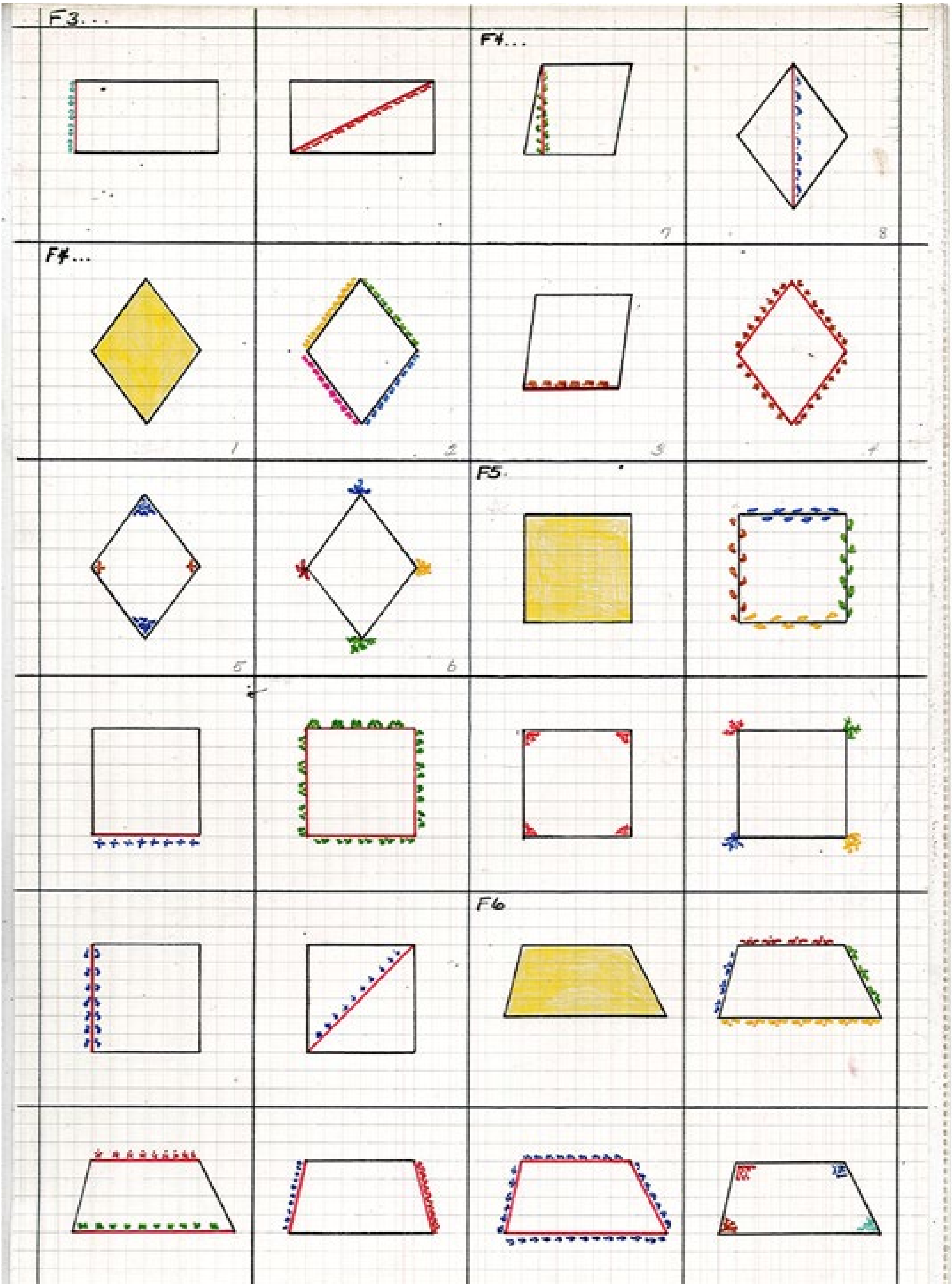
All cards 13.8² | Scale 12/13.8 | 1 square = 3/4 cm. | ②

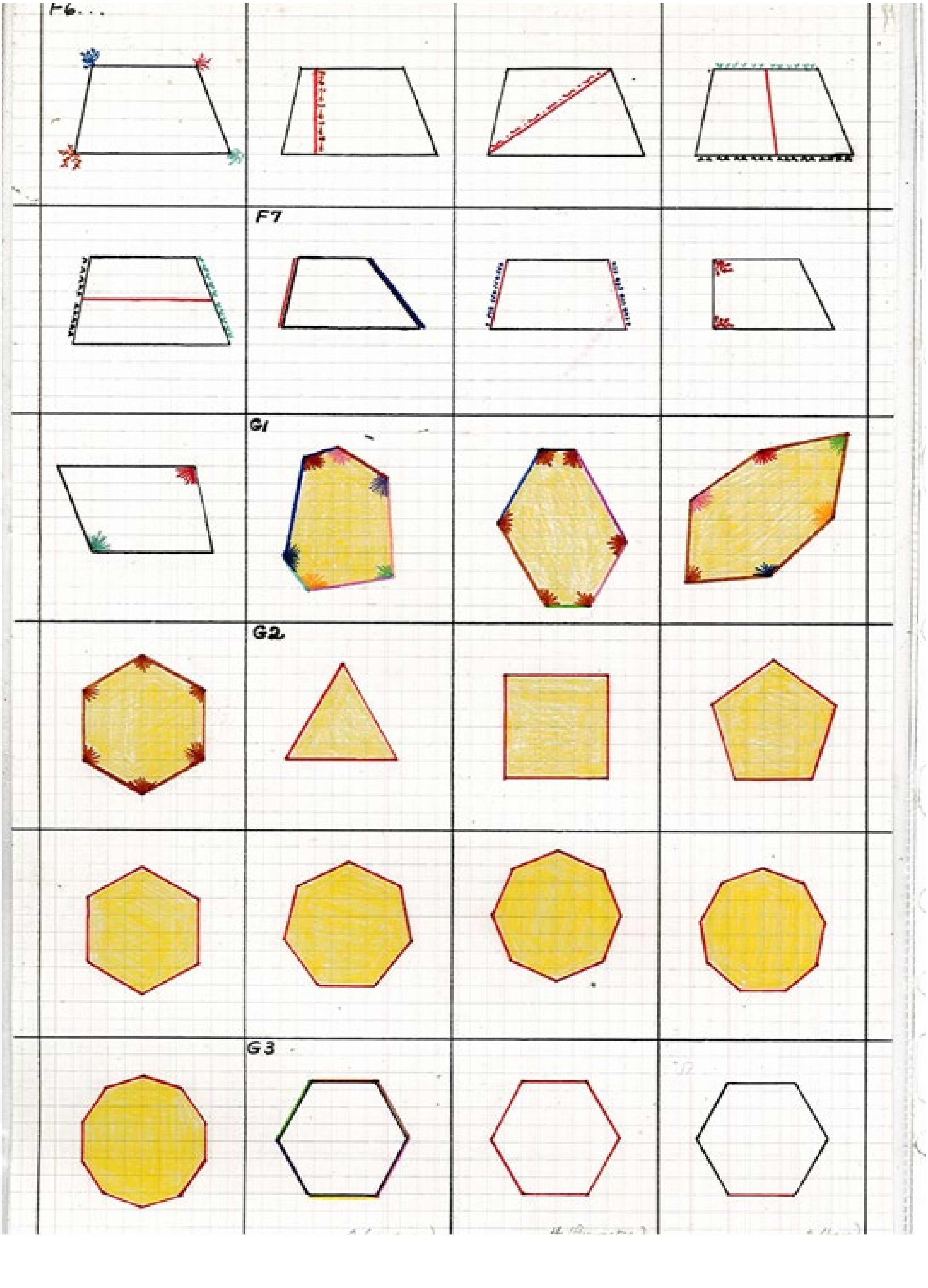


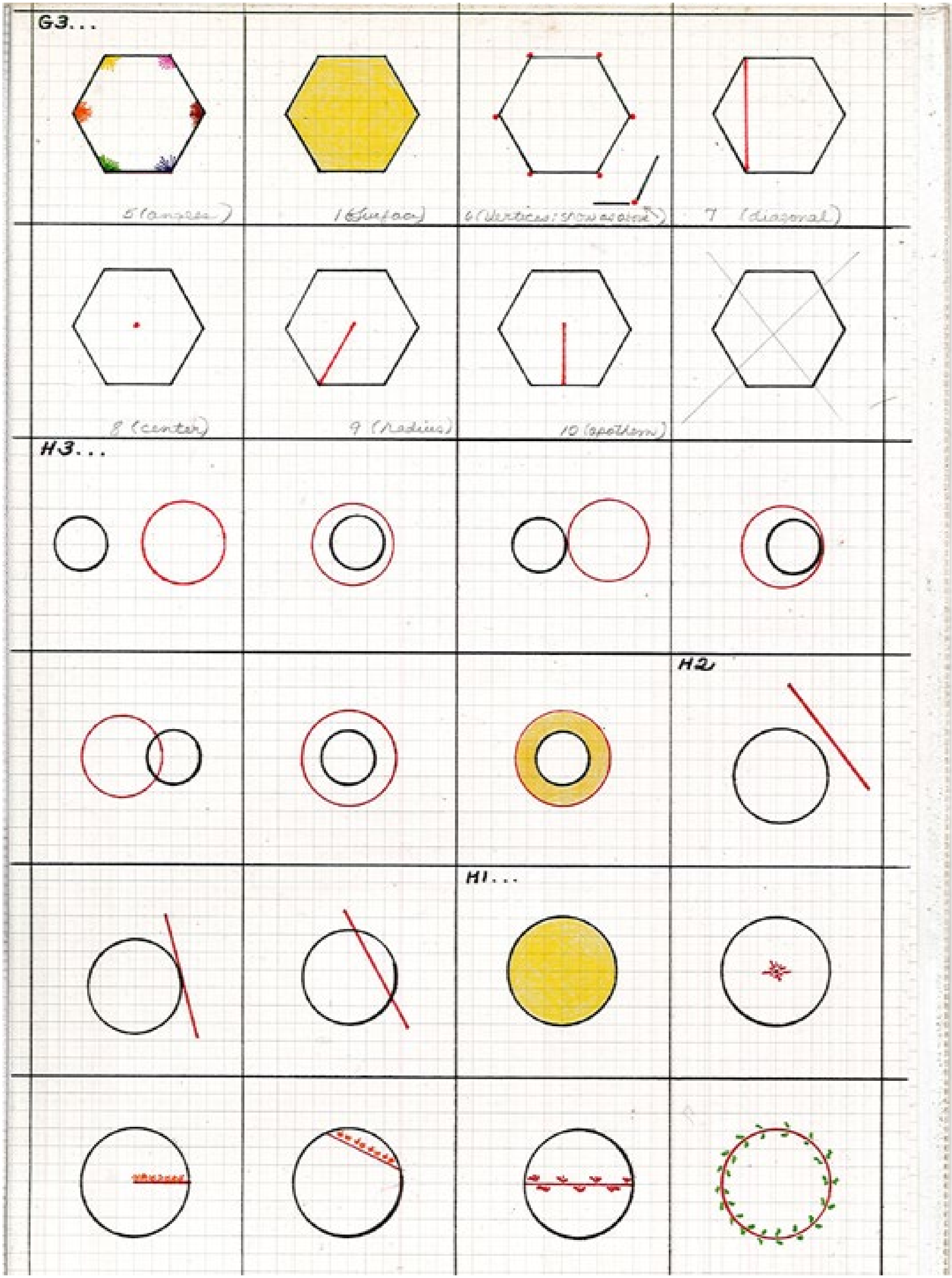

C3
1
3
2
C4
C5
C6
C7


D1
D2
D3
E1
E2
E3

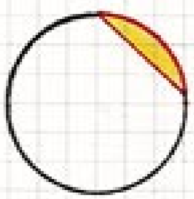

E4
E5
F1 II
F2
F3


F3...
F4...
7
8
F4...
1
2
3
4
F5.
5
6
F6


F6...
F7
G1
G2
G3


G3...
5 (angles)
7 (diagonal)
8 (center)
9 (radius)
10 (apothem)
H3...
H2
H1...

H1. . .



A1 Booklet contains 2 additional picture cards:

(point)

(line)

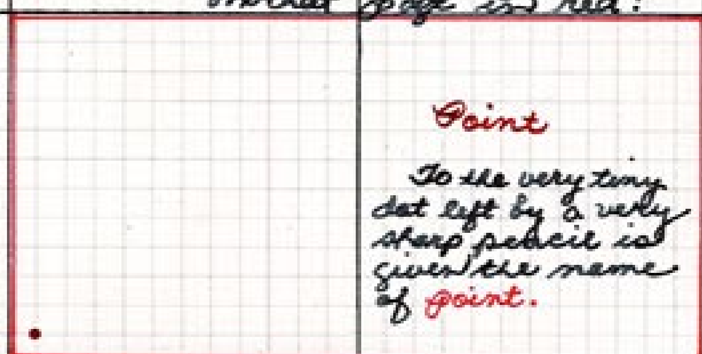
and a solid red cardboard square (surface)...

and the fold-up red cardboard cube (solid).

The ~~booklets~~ folders contain: picture cards
labels (giving names)
definitions:

"To the very tiny dot left by a very sharp pencil is given the name of...

The booket is composed of: picture cards on the left side and definition on right. Name given on that page is red:



SECOND LEVEL: Classified Nomenclatures. . .
Materials. . .

NOTE: The wall charts are stored in a long wooden box, each marked with the series letter and a number. These letters and numbers correspond to those found on the booklets. The booklets and folders are stored, in order, in a series of wooden holders.

The 8 series:

A) Basic Ideas. The concept of point, line and surface and solid. Gold: honey orange. An important color, for it represents the color of the decimal system. These four concepts constitute the base of plane and solid geometry, the beginning of the whole geometrical world.
B) The Study of Lines. A preparation for successive series. Light green.
C) The Study of Angles. A preparation for successive series. Yellow.
D) The Plane Figures. To define plane, we must have the preceeding concepts of line and angle. Here there is a consideration of two kinds of figures: those limited by curved lines and those limited by straight line segments. The distinctions between all figures defined by curves --- and all the polygons. Here we have begun our study of those figures found in the geometry cabinet. The successive series follow the pattern found in the cabinet. Pink.
E) The First Polygon: The Triangle. Grey.
F) The Quadrilaterals: Nocciola.
G) Regular Polygons. Red.
H) The Circle. Bright green.

Presentation: Pattern valid for all the nomenclatures: here presented the Classification of the Triangle According to the Sides.

NOTE: It is important always that we work first with the materials; the nomenclature is what is left afterwards; that with which the child works. It is not the lesson.

1. Take from the folder the picture cards and display them on the mat. Use the three-period lesson for review if necessary.

2. Distribute the reading labels. The child reads them and matches each with the corresponding picture cards.

3. He checks his work with the wall chart. CONTROL.

End of the work on the first level of reading words:
If the child reads sentences, proceed. . . . .

4. With the picture cards displayed and the labels shown on the card, the definitions are distributed and the child reads it, then places it below the picture card and moves the label down onto the definition at the place in the sentence where it fits.

OR

The child simply uses the definitions and the labels. He reads the definition and then finds the correct label, showing it on the dotted line in the definition.

5. CONTROL. The booklet. In the first method of #4 we have constructed, page by page, the booklet. In the second, here the child sees the picture. The elementary child usually works with this second, shorter exercise.

NOTE: When the material is first introduced, the folder contains only the picture cards and the labels and the wall charts are used; when the child reads sentences, the definitions and the booklets are added.

An EXERCISE: The child writes the definition, then cuts it into pieces, mixes the pieces up and rearranges them in order. He may do several at one time for the added interest of separating the correct sentences.

NOTE: In the casa material, there is an additional card in the folder called the "Spoken Design," a card with the illustration and the word written on it.

THE BOX OF STICKS: Parallel work with the Classified Nomenclature

Material

1. A wooden box with divisions to contain the following materials.
2. Sets of 10 sticks in ten different lengths and colors.
    - a) Brown. 2 cm.
    - b) Purple. 4 cm.
    - c) Orange. 6 cm.
    - d) Red. 8 cm.
    - e) Black. 10 cm.
    - f) Beige. 12 cm.
    - g) Bright green. 14 cm.
    - h) Pink. 16 cm.
    - i) Blue. 18 cm.
    - k) Yellow. 20 cm.

    SO. . .there is two sensorial stimuli for the recognition of the sticks: both length and color. The color is not significant: it has no connection with the bead colors. However, the slender wooden sticks have significant measurements. Each one increases by 2 centimeters. The length is from one small hole at one end to another small hole at the other end. (There is a distance of 6 mm. from the hole to each end) 2 or 3 of the sticks in each series have holes along the sticks, equidistant.
3. 6 semi-circles, small --- medium --- large, in couples. They also have holes at each end of the arcs.

Is it possible to construct, with these sticks, all the triangles? NO

Given the sides of the right-angled triangle, scalene or isosceles, if we know the measurements of the two legs, what will the length of the hypotenuese be? Pythagorus figured it out with his triangle of 3, 4, and 5. The sides of his triangle, then, were designed to be even, but the resulting hypotenuese was 5, an odd number.

We note the theorem:

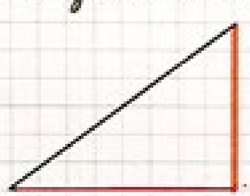
$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = 5^2$$
$$9 + 16 = 25$$

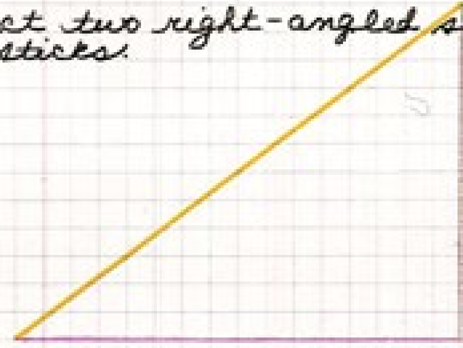
Since the holes on our sticks are equidistant and all represent even quantities, we can't construct this important original triangle which Pythagorus devised.

So we must look to other triangle measurements for our construction using the Pythagorean theorem. We discover two possibilities:

$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = 5^2$$
$$\times 2 \quad \boxed{6^2 + 8^2 = 10^2}$$
$$\times 3 \quad 9^2 + 12^2 = 15^2$$
$$\times 4 \quad \boxed{12^2 + 16^2 = 20^2}$$
$$\times 5 \quad 15^2 + 20^2 = 25^2$$

Thus we can construct two right-angled scalene triangles with our sticks.



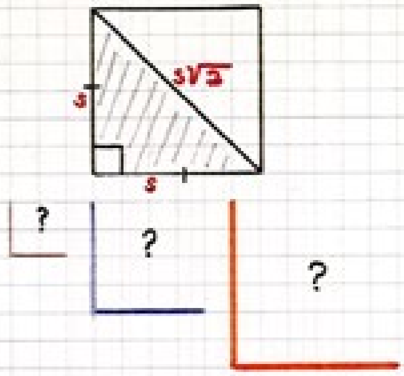


THE BOX OF STICKS. . .

Material. . .

The right-angled isosceles triangle is always half of the square whose side is s. Then the two equal sides of the isosceles triangle are s = s.

Do we have the sticks to construct the hypoteneuse of this triangle?



NO. Because the sticks are all even lengths, s will always be an even number. AND the diagonal of the square which is the hypoteneuse of the isosceles triangle is equal to $s\sqrt{2}$ that is, the side taken as many times as the square root of 2 which is an irrational number. (1.4142. . . . .)

And because this hypoteneuse will always be an irrational number, we do not have the sticks which will satisfy this measure.

Therefore,

4. A second box is included in the material containing 10 sticks, one for each of the possible isosceles triangles which we can construct with the sticks. So the measurements of these ten sticks will correspond to the hypoteneuse of each of those triangles, that is, for

s = 2	hypoteneuse stick will be	$2\sqrt{2}$	$= 2 \cdot 1.4142...$
s = 4	hypoteneuse stick will be	$4\sqrt{2}$	$= 4 \cdot 1.4142....$
s = 6	hypoteneuse stick will be	$6\sqrt{2}$	$= 6 \cdot 1.4142...$
s = 8	hypoteneuse stick will be	$8\sqrt{2}$	$= 8 \cdot 1.4142...$
s = 10	hypoteneuse stick will be	$10\sqrt{2}$	$= 10 \cdot 1.4142...$
s = 12	hypoteneuse stick will be	$12\sqrt{2}$	$= 12 \cdot 1.4142...$
s = 14	hypoteneuse stick will be	$14\sqrt{2}$	$= 14 \cdot 1.4142...$
s = 16	hypoteneuse stick will be	$16\sqrt{2}$	$= 16 \cdot 1.4142...$
s = 18	hypoteneuse stick will be	$18\sqrt{2}$	$= 18 \cdot 1.4142...$
s = 20	hypoteneuse stick will be	$20\sqrt{2}$	$= 20 \cdot 1.4142$

The sticks are a natural wood color and each one has small holes 6 mm. from each end. The measurement is that between holes.

5. Two pairs of scissors.
6. Envelopes.
7. A box of "Geometry Supplies for the Stick Box containing:
   a) tacks of different colors.
   b) nails with red plastic heads.
   c) paper holders
   d) three crayons: red, blue, black.
8. A small hammer. (type used by watchmakers)
9. A large wooden board covered by several pieces of paper which are pegged with push pins at each corner. This represents the geometric plane. When we place a stick on it, we say "this is a line segment lying on the plane."

NOTE: The materials here described are the tools for the work. The nomenclature provides the projects. It acts, too, as a teacher for the child, giving help when needed and guiding his work.

THE BOX OF STICKS. . .

THE STUDY OF THE GEOMETRY NOMENCLATURE

Material

1. A box. (rectangular prism)
2. A ball.
3. The plane insets. (the geometry cabinet)
4. The series of small solids. (light blue in a wooden box)
5. Paper.
6. A red pencil, well-sharpened.
7. A pencil sharpener.
8. From the decimal system material: 10 units, 10 ten-bars, 10 hundred-squares, 1 thousand-cube.

NOTE: In casa the small solids were used with the direct aim of addressing the stereognostic sense. (stereognostic meaning 3-dimensional, lost knowledge). Solid geometry is sometimes called stereometry. In the children's house work, the child feels the solids in a bag and knows the shape well. In the elementary school, the child does not know this material, but he knows the shapes: his ice cream cones have always been conical-shaped figures.

DIRECT AIM: To give the basic ideas of plane and solid geometry---that is, point, line, surface, and solid---based on reality, on the decimal system materials.

Presentation

1. Invite the child to place before him the small box. Then give him the ball, asking him to place the ball in the exact place of the box.

CONCEPTS: The ball and the box, each representing a body, occupy space.
The ball and the box are opposite figures: one limited by straight lines, representing a **plane** surface; and the other limited by curved lines, representing the image of a curved surface.

1. I didn't ask you to remove the box. . . nor to put it on top of the box. . . neither to put the ball to the side of the box or behind the box. I asked you to put the ball exactly where the box is!

But you are crazy.

This means that each body (matter), each thing, occupies space.
Each thing occupies a place.
(Thing and Place a better nomenclature for the child.)
This is so true that when I want to go out I must ask you to move if you are in the way.

Imagine how many things there are to occupy a place!!

2. Introduce the small solid insets.

CONCEPTS: To introduce various solids each of which occupies space.
To emphasize the idea of a solid.

The solids are displayed on the mat. Use the small wooden holders for the sphere, the ovoid and the ellipsoid.

2. This was my ball. Now I can call it the sphere.
This was my box. Now I properly call it a prism.
Look, this has a curved surface like the sphere and a straight side like the prism. It is called a cone.
The sphere and the prism represent the extremes of each. We are filling in the variations between the two.
This one also has a curved surface and two plane parts of the surface.
It is the cylinder.
This is the cube.
This is another prism.
Here is a pyramid.
And another pyramid.
This looks like an egg---it is a ovoid.
It is all curved.
And so is this one---the ellipsoid.

3. Give the nomenclature for the prisms and pyramids.

3. Let's give a more precise name to each of these solids.
This is a <u>triangular prism</u>. . .and this is a <u>quadrangular prism</u>.
This is a <u>triangular pyramid</u>. . .and this is a <u>quadrangular pyramid</u>.

4. Present again the ball and the box, examining the "surface."

Then take the figures in the first drawer of the geometry cabinet and note the surfaces. Use other drawers to expand the idea.

CONCEPT: Giving the concept of surface.

4. This time let's examine how this object is limited: the box.
It is a plane surface and stable.
The circle has a curved surface and is not stable. Watch it roll.
What is the surface?
It is like a very very thin layer of paint which covers the outside of a thing.
It might be compared to very thin tissue paper covering the object.
Look at these figures---the insets.
The surface is the thin layer of paint.

5. Showing the square and the circle, note the different lines which limit the figures.

Then, with paper and pencil, draw the line.

CONCEPT: Giving the concept of line. The concept of the thin layer of surface is given here as well as the thinness of the line.

5. Is this a plane surface? (square)
How is it limited?
By a line.
What kind of a surface is this? (circle)
It is limited, too, by a line; but a different kind of line.
We have here two regions limited by different lines.

Look---we can draw a line.
Let's sharpen the pencil well---now it is very sharp.
This can be the image of a line.
But this is very thick.
It should be much much thinner.

6. Now with the pencil, the point again sharpened, make a point.

CONCEPT: Giving the concept of point.

6. Now let's sharpen our pencil again.
We want to make a very very small pencil point.
This is the image of a point, but it should be much much smaller.
The point is the mark left by a "very very sharp pencil point."

NOTE: We approached here the concepts of point, line, surface and solid in reverse order, giving the solid first because the child believes in reality. And so he can more easily understand the 3-dimensional which composes reality. We move from that concept to those which have no dimensions.

Presentation #2: Using the Classified Nomenclature

1. From one of the envelopes of the classified nomenclature, for #A1, take the red collapisible cardboard cube and fold it into a cube. Then take the red square and the card on which is drawn a line and finally the card which shows a point in the middle. Give the <u>nomenclature</u>.

1. This is a solid body.
This is a surface. It is all red----it goes on to infinity even though here it is cut.
This line also goes on indefinitely at both ends.
And this is a point.

Presentation #2. . .

2. With the child, try to formulate definitions for these four ideas of point, line, surface and solid.

NOTE: Mathematicians call these entities. In our definitions for the first three we have used "is like," because the point IS NOT a mark, the line IS NOT a hair, the surface IS NOT a square. With the solid we have not said "is like" because the solid is three-dimensional reality.

2. A solid is what occupies a place. A surface is like a square or a circle. It can be plane or it may bend and be curved. The line is like a very thing hair--- it can also be straight or curved. The point is like a very very thin mark made by a well-sharpened pencil point.

3. The child matches the labels to the four entities shown.

4. Second and third period lesson.

Presentation #3: Point, line, surface and solid with the decimal materials.

To emphasize these basic ideas, we use the decimal system materials. With this material we represent the four basic concepts.

1. Display, one at a time, the unit bead, 1. the ten-bar, the hundred-square and the thousand-cube. As they are laid on the mat, give the familiar names. Then identify the four as the four entities.

This is the point, the bar, the square and the cube.
Now this is the point,
this is the line,
this is the surface,
and this is the solid.
We have named these four pieces with geometrical nomenclature.

2. By adding nine beads, one by one to the first, show that the series of points makes the line. Then add bars to show that the series of lines constitutes the surface. Finally, add squares to the one to show the composition of the solid. Always add 9 pieces.

CONCEPTS: The origin of the solid lies in the surface, the origin of the surface lies in the line, the origin of the line lies in the point. . . .AND If we analyze the 4 statements, we find that the point is the constructor or all.

Here is one point. I add one more point, one more point, one more point. . . . . By adding points we have formed a line. We can say that the point is the constructor of the line. Now we take the line and add one more line. . .and one more line. . .and one. . . It is a surface. (As soon as we have two lines we have a surface, but we have used ten to have the same shape as the surface of the square.) So the line is the constructor of the surface. Now we take this surface. . .and one more. . .and one more. . . And we have constructed a solid body. It is evident that the surface is the constructor of the solid.

OR
WE CAN THEN
SHOW

2. Using only one piece of each material, that it can simply be moved to create the next.

2. I can move the point to create the line. Have you ever seen the tip of a lighted cigarette in the dark. It makes a line. But when it stops moving, it is a point again. We take the line and move it to form a surface. The line can be moved in different ways. In the line we can still see the point. . . and it is the line that created the surface.

THE STUDY OF THE GEOMETRY NOMENCLATURE. . .
Presentation #3. . .

2. . .the second demonstration of the construction of the line, surface and solid from the single point.

2. .Moving the surface, we create the solid, but we can still see the line and the point. And we can see that the point is truly the original constructor of all.

NOTE: An interesting demonstration of this is with a broad brush which can, with different stroking, show the progression of all four in an interesting way.

3. Present the Classified Nomenclature #A1, showing first the picture cards of point, line, surface and solid. (The point here shown in the lower left corner, precisely placed so that the successive cards may all begin with this same point and still lie within the limits of the card). The child matches the labels, uses the definitions. The control booklet is shown and the wall chart utilized now in the classroom.

Age: 6 years.

THE NOMENCLATURE STUDY: THE LINES

Material
1. From the plane insets of the geometry cabinet, the first and sixth drawers.
2. The second series of picture cards from the cabinet series: contours only.

Presentation: The technical level of the work; discovery with the plane insets

1. From the first drawer, take the square, then the circle and trace the contours with your fingers, identifying the limiting lines. Give the names for the lines.

1. This is a curved line.
This is a straight line.

2. Repeat with the insets of the sixth drawer.
NOTE: Both drawers give a good contrast of the curved and straight lines.

3. Take the second series of picture cards used with the geometry cabinet---those with only the thin line contour. The child forms two columns, one of the figures limited by straight lines, one of those limited by curved lines. Then the child makes a list of those two columns.

Presentation: The pure concept isolated from the object

Material
1. A string, wound at both ends onto a small cardboard spool. . .about two meters in length.
2. The box of sticks.

NOTE: The exploration of curved and straight lines is linked directly to the figure. It is, however, necessary to go into the concept of line independent of the figures which the line limits.

1. With the spools held one in each hand, show a small stetch of the string, and then gradually lengthen the string stretched between the hands, showing the "line" in different spatial positions, THEN showing it as a straight line and a curved line in various ways. The string continues to grow longer and change from straight to curved.

1. This is a line. . .Look. . .
this is a line. . .this is a line. . .
This is a straight line. . .
this is a straight line. . .
this is a straight line. . .
This is a curved line. . .
this is a curved line. . .
this is a curved line. . .
This is a straight line. . .a curved line.
The line is changing its position in space.

THE STUDY OF THE GEOMETRY NOMENCLATURE. . .
Presentation: The Lines: The Pure Concept. . .

2. Ask for a volunteer to draw a line. Note that it is impossible without a qualification. Show how the line can be written as straight and curved with dotted lines at each end.

2. Who will come and draw a line?
It is impossible to draw a line because we have not given it a quality.
"A Line" is without qualities.
Before drawing it, one has to establish whether it is curved or straight.
We can write a straight line like this:

Or a curved line:

3. Three period lesson.

3. What is a line when it looks like this?

4. Ask the child to choose a straight and curved line from the box of sticks. (These are actually line segments, which we will discover later)
NOTE that the line has no end.

Show me a straight line here. . . a curved line.
These each look as though they had ends.
With our string, my hands made a limit to the line, but actually the line has no end.
Our hands create a string that has no end.
When we speak of a line, when we identify a line, with a stick or anything else, we must imagine that it goes on indefinitely at both ends.

5. The child draws the lines, straight and curved.

4. That is why, when we draw a straight or a curved line, we must imagine that it goes on indefinitely by writing dots at the beginning and end to show that it goes to infinity.

A GAME: The investigation at the level of reality, the environment itself

Ask the child to look for straight and curved lines in the environment. He makes a list. (The environment is really made of line segments---but later we will define more accurately the qualities of the line and its parts)

GRAMMATICAL ANALYSIS: Analyze with the child the grammatical expressions:
the line
the straight line
the curved line

Identify the parts of speech, then the child copies the phrases and draws the corresponding symbols OR he matches the symbols with slips of paper on which the phrases are written.

With THE GRAMMAR BOXES, prepare the phrase "the line" for the noun box and the second two phrases for the adjective box exercise in which he matches the two distinguishing adjectives with one noun and article.

NOTE: In Italian, the phrases can be reduced only to the article and the adjectives "straight" and "curved". . .which become nouns. . .or take the place of the nouns. It is then sufficient to say only "draw the straight" or "draw the curved."

THE NOMENCLATURE STUDY: THE POSITIONS OF A STRAIGHT LINE

Material
1. The sticks in box and all necessary supplies.
2. Two beakers.
3. Red coloring: powder or liquid.
4. A spoon, a cloth.
5. The globe.
6. Water in a pitcher.
7. A plumb line.

THE STUDY OF THE NOMENCLATURE. . .

The Positions of a Straight Line. . .

Presentation

1. Put a small amount of red coloring into the pitcher to color the water. Then pour about 4 cm. of water into each beaker.

2. Move one of the beakers to stir the water; and then wait until the water is again as still as the water in the second one, our point of reference. (An exercise in patience)

3. Give the term horizontal and its etymology.

3. Now the water here is as still as the water here. We say that the position of the surface of the water is horizontal. The word comes from "horizon," that imaginary line that divides the earth and the sky.

4. Take a red stick and lay it carefully on the surface of the water; wait again until the water is still.

4. This is a straight line. Let's see what position it takes. The position of this line, this straight line, is horizontal. It is the horizontal straight line. It is lying on the surface of horizontal water.

5. Taking the stick out of the water now, line it up with the water's surface outside the beaker.

5. What is the relationship of the position of this stick to the water? Now I have understood when the line is horizontal. I know that the surface of still water gives me the image of the horizontal surface. So the stick is a horizontal straight line.

6. Invite the child to hold the plumb line, noting the red color of the string as an emphasis of the quality we want to show. Give the term vertical.

6. Look---this plumb line has a red string. We'll wait until the plumb line is completely still. This is the image of a straight line in a vertical position.

7. Hold the red stick alongside the line and identify its position.

7. This straight line follows the position of the string. . .so it is lying on the vertical plane. It is a vertical straight line.

8. Show the plumb line as that line which passes through the center of the earth, using the globe as reference and showing that it can pass through from one of an infinite number of points---vertically. (Don't confuse it with the axis by showing the line passing from pole to pole.)

8. Where are we on the globe? The plumb line, the vertical straight line, also is the line that passes through the center of the earth. It can pass from any point through the center to the other side---and we call it the vertical line.

9. Moving the red stick between the two planes, identify oblique as that countless number of gradations between the two positions.

9. When our stick is this way, this is horizontal. It is only one position. This is vertical. . .only one position. BUT in between there are an infinite number of positions intermediate----as we go from one to the other. A straight line, when it is not horizontal or vertical, is an oblique straight line.

THE STUDY OF THE NOMENCLATURE. . .
The Positions of a Straight Line. . .

A GAME: Using the red stick in a variation of spatial positions, have the child say the word for the straight line position as the stick moves: horizontal, vertical, oblique. He discovers that there is only one position where the first and second terms are used, but "oblique" is repeated an infinite number of times in between the two.

. . .horizontal. . .oblique. . .oblique. . oblique. . .oblique. . .oblique. . .oblique. . . .oblique. . .oblique. . .vertical.

Presentation: THE POSITIONS OF A STRAIGHT LINE: On the Plane

Now we must transfer the concepts from space to the plane, to prepare the child for his work.

1. Show the wooden board, covered with the pieces of paper. Identify.	1. This is our plane.
2. Take out of the box 3 red sticks (8 cm.) and fix one to the board, in a horizontal position. Identify that position.	2. This is a straight line. Look at its position. What is it?
3. Now moving the whole board, identify the changing positions of the stick. Move the board through all the planes, repeating the single word identification game.	3. Now tell me the position of the straight line. . .now. . .now. . .now. . . We cannot go on working like this. We have to decide something.
4. Show the board up and down, identifying vertical---then turn it so that the stick is horizontal.....finally, in the same plane, show the oblique position and identify.	4. This straight line that looks like a plumb line is called vertical. When we look at something in a vertical position, we look up and down.
	When the straight line is in the position of still water, we look at it from left to right as the horizon looks---and the straight line is horizontal.
Show a number of oblique positions.	When it is not in either position, it is oblique. . .it is oblique. . .it is oblique. . .it is oblique.
5. Fix the other 2 sticks to the board, showing the three positions now. Three-period lesson.	5. Which straight line is in the vertical position? How is this one?
6. Tell the child that we can now define the three sticks in relationship to the plane. And we can draw them on the plane. NOTE: The board is horizontal.	6. If we consider these three sticks in relation to space, they are all horizontal. BUT IN RELATIONSHIP TO THE PLANE, THEY ARE ALL IN DIFFERENT POSITIONS. Now you don't need to draw a vertical line in the air---you can draw it on paper.
7. Introduce the nomenclature picture cards, noting that they are drawn on the plane. The child matches the cards with the three sticks on the board.	7. The straight lines on these cards are drawn according to the plane. Notice the dotted lines on each of the sides. Remember what that shows?

A GAME: Looking for lines in the environment in the three different positions.

THE STUDY OF THE GEOMETRY NOMENCLATURE. . .
The Positions of the Straight Line. . .

A GAME: GRAMMAR ANALYSIS

Now adding the qualities "horizontal, vertical and oblique" we can analyze with the child the new phrases: the straight horizontal line
the straight vertical line
the straight oblique line.

Again the child may identify the words, match with the symbols AND the grammar boxes may be used----the adjective grammar box with the exercise of several adjectives with the same noun.

NOTE: In Italian, we see the same deletion of words so that the phrase becomes simply "the horizontal, the vertical, the oblique."

Presentation: THE PARTS OF A STRAIGHT LINE

Material: the same. . .add two pairs of scissors

1. Take again the spools of the string and draw it out between the hands. Give the two spools to the child to hold the string.	1. This is a straight line.
2. With a red crayon, choose a point on the string, mark it and then cut at that point.	2. I am going to choose a point on this string. I'll mark it with the color of fire to show that specific point. NOW. . . This is a ray. . .this is a ray. . . The red point is the origin. This was a straight line. When it was cut at the point, we marked the beginning. The ray has an origin, a beginning at one side and no end on the other. That second side goes on to infinity.
3. The child again holds the string by both ends. This time mark two points and cut simultaneously with two pairs of scissors. Define line segment.	3. Now show me a straight line. This is a straight line. This is a ray. (marking one point) AND. . .this is a line segment. The line segment has two end points. . . there is a beginning and an end.
4. Show that the sticks in the box are line segments.	4. Now I understand that the sticks in the box are not straight lines but line segments because each has a beginning and an end. And it is evident that when we curve one, we have this----an arc with a beginning and an end.
5. Show the child how to draw the three parts defined.	The line goes on to infinity on both ends. The ray goes on to infinity on one end. The segment doesn't go anywhere---it has two specific ends.
6. Present the nomenclature cards, labels, definitions, booklet, wall chart: always the final part of the pattern.	

PARALLEL, CONVERGENT AND DIVERGENT LINES: Two Love Stories and the Story of Indifference

Material

1. Two plumb lines with red strings.
2. The wooden plane, the box of sticks and the geometry supplies.
3. An envelope containing: small figures of three pairs of children:
   a happy pair
   a sad pair
   an indifferent pair
   and : three pairs of arrows, red plastic, about 3 cm. long:
   two pairs with arrows on one end of the line
   one pair with arrows pointing in both directions.

Presentation

1. AN IMPRESSION: Ask two children to hold the plumb lines. When they are still, measure the distance between them at several points. Ask one child to step back a few steps. Wait until the lines are still. Measure the distance at several points again. Introduce the word parallel and concept.

1. We must wait until both lines stop moving.
New let's see what the distance is between these two red lines: it is the same here at the bottom and at the top.
What is the distance between the lines now?
The distance between the two straight lines is the same here. . .and here. . . and here.
When we have two straight lines the same distance apart at every point, those lines are parallel.
We can imagine that these two lines go into infinity in both directions---up and down. . .and never would they meet.

2. Give the etymology of the word.

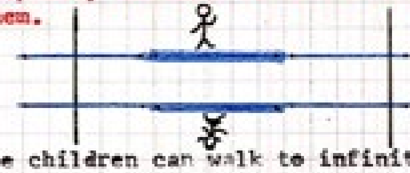
2. The word "parallel" comes from a Greek word "parallelus" meaning "beside one another."

3. ON THE PLANE: Introduce the three pairs of children and the arrows.

3. Today we are going to tell the stories of these three couples of children. These first two children look very happy---and these two very sad. Their stories must be quite different. And these two children seem to have almost no expression at all. We'll use these arrows to help tell the stories.

4. Construct the parallel lines on the plane: take any stick, a longer one (yellow) and fix it to the board. Then take two guide sticks, the two 4 cm. purple ones, and place them down vertically from the first stick with the two ends touching it. Then put a second stick horizontally (the same color as the first) to rest along the other two ends of the guide sticks and thus become parallel.
Remove the guide sticks.
Show the indifferent children on the exterior of each line, walking along.

4. Here we have the story of indifference. These two straight lines are like two children who are no longer friends. They walk along and they do not wish to meet, nor do they wish to increase the distance between them.
These lines are parallel because, no matter how far they extend into infinity they keep the same distance between them.

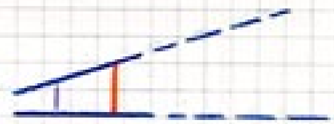


5. Lay two more sticks on each side, prolonging the lines off the plane, first in one direction and then in the other. The concept of infinity.

5. The children can walk to infinity and they will never meet. <u>They can walk to infinity in either direction and they will never meet!!</u>

6. Substitute the two-directional red arrows for the children. First we might substitute the one-directional arrows---two above, showing the two directions and two below. THEN use the two-directional arrows in place of the four. REMOVE THE FOUR EXTRA STICKS---the arrows now give the idea of infinity.

7. Show another similar stick in the left-hand quadrant of the board---placed horizontally also. CONSTRUCT the DIVERGENT LINES by using one of the previous guide sticks and one of the next measurement---orange (6). Place them above the horizontal stick, running vertically and then place the second stick, also equal to the first three horizontals, along the second ends of the guide sticks. Remove the guide sticks.



NOTE: The word divergent was invented in 1611 by Keplero simply to provide a name for the opposite of convergent.

8. Show the sad children walking these lines, headed away from each other.

8. These two straight lines are divergent. These two children were friends before, but they had an argument and they do not wish to meet again. The distance is always increasing between them.

9. Replace the children with red arrows pointing the divergent direction; add other sticks, prolonging the lines off the plane---to infinity. Then remove the additional sticks.

9. The longer these lines prolong, the greater the distance will be between them.

10. Lay one stick, equal to the other four now shown on the plane, in the right-hand quadrant of the board, also horizontally. Construct the CONVERGENT LINES by using the same guide sticks, but in reverse order; that is, with the shorter one second.

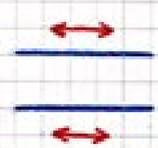
11. Show the happy pair along these lines.

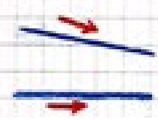
11. These two straight lines are convergent. And this is a true love story. The two children see each other from far away and want to be closer and closer.

12. Replace the children with converging arrows: extend the lines off the plane, showing how they cross and become divergent.

12. But, like all love stories, there was a bad ending. There was a point where the two met and then they began to move apart again. At this point of crisis the convergent point becomes the divergent point. And the convergent lines become divergent lines.

13. Three-period lessons: Which lines are parallel? Tell me how these lines are.



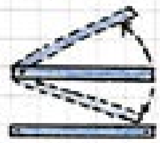


14. Remove one pair (either convergent or divergent lines). Repeat the three-period lesson, switching the direction of the arrows on the second set to show both convergent and divergent lines.

14. Now we have only two pairs of lines. Show me the divergent lines. . .
the parallel lines. . .
the convergent lines. . .
It is enough to change the direction of the arrows.

15. Use only one pair of arrows, not fixed to the plane, and the two arrows (one-directional). Do the three-period lesson, asking the child to construct the parallel, convergent and divergent lines with these two sticks and the arrows.

16. Fix one stick to the plane and then, using the two equal guide sticks, fix the second stick by one end only. This is the last simplification. NOW IT IS NOT EVEN NECESSARY TO USE THE ARROWS. The child can show all three positions of the two lines by moving the second stick up or down or showing it parallel.



THE STUDY OF LINES. . .

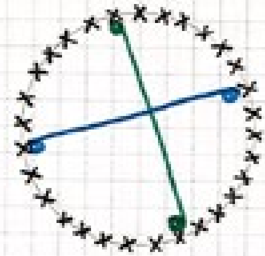
PERPENDICULAR AND OBLIQUE LINES

Material

1. Two balls of different colored yarn, prepared so that each ball is wound in two balls loosely.
2. The plane, the box of sticks and the geometry supplies.

Presentation

1. AN IMPRESSION: The children sit on the line, forming a circle. Sit in the middle and roll one ball of the first yarn in one direction---to one child. Then roll the other end to a child in the opposite direction. Do the same with the second yarn, passing its two balls to children opposite each other and in the other four quadrants.



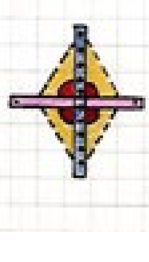
2. Note that there are now four angles formed formed on the plane---the floor. THE CHILDREN MUST STRETCH THE YARN TIGHTLY BETWEEN THEM TO SHOW THIS. And that the angles are formed by four infinite rays.

2. If you had missed the ball of yarn when I rolled it to you, it would have gone on and on---to infinity.
We have shown four rays.
Where is the point of origin for each?
How many angles have we formed?
The rays have divided us into four groups of people---each group within one of those four angles.
Are we four equal groups? NO

3. Pass one color yarn (both balls), to the left on both sides of the circle. One person at a time on both sides of the line until the four groups, the four angles, are equal. Describe the changing character of the groups. Give the nomenclature---perpencidular and oblique. Then stop the one color and move the other. The children say the nomenclature as the one moves. . . stopping at perpendicular.

3. Now we can pass two of our rays---those the same color to change the size of the angle.
These two rays form a straight line and it is crossing another straight line to form different sizes of angles.
How many angles are there now? now?
Are they equal angles? NOW---YES
When the four angles formed by these two straight lines are equal, the lines are perpendicular.
When the lines are perpendicular, the four angles are equal.
Now let's pass the same colored balls again.
When the four angles formed by these two straight lines are not equal, the lines are oblique.
Now tell me how these straight lines are.
Oblique. . .oblique. . .oblique. . .
Perpendicular.

4. ON THE PLANE: Fix one stick horizontally on the board. Take another, longer stick (one with holes all along it) and fix it perpendicular to the first, showing the four equal angles by measuring with the measuring angle.

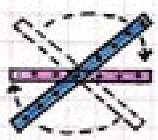


4. In order to show perpendicular lines on the plane here, we must show these two straight lines forming four equal angles.
We know that with one straight line we show a straight angle.
Then we will place the second straight line so that it forms two equal angles--two right angles with the first.
And when we have formed two equal angles here, we see that we have formed four equal angles.
Let's measure each one to be sure.
Then we have shown perpendicular lines.

5. Etymology:

5. "Perpendicular" comes from the Latin "perpendiculum," meaning "plumb line," or, more specifically, "an instrument handing through."
We can imagine this perpendicular line as a plumb line hanging through the horizontal line---or the horizon.

6. Using the same two colors of sticks, fix one horizontally below the first set. (IF BOTH STICKS USED HAD HOLES ALL ALONG THEM, THE SECOND STICK CAN BE FIXED AT ABOUT CENTER). Show the second stick across the first in an oblique position. With the measuring angle, show that the angles are not equal. The second stick then moves to create various situations of the oblique position.



6. Let's experiment with the angles that these two sticks form.
Let's use the measuring angle to see what one of the angles measures.
This is not a right angle.
It is enough to measure this one angle to know that the other three are not equal.
Then these two are oblique lines because they do not form four equal angles.
How are the lines now?
Measure an angle to find out.
Now? oblique. . .oblique. . .oblique. . . perpendicular. . .oblique. . .oblique. . .
When two straight lines are not perpendicular, they are oblique.

7. Etymology:

7. "Oblique" is a word that comes from a Latin word meaning "to move towards."
As we move these two straight lines, what are we moving towards?
It also comes from a word meaning "elbow."
Can you imagine the oblique lines that are formed if you could extend the lines that your elbow makes when it is slightly bent?

8. Three-period lesson.

9. Classified Nomenclature materials introduced.

THE STUDY OF THE GEOMETRY NOMENCLATURE. . .
THE STUDY OF LINES. . .

TWO STRAIGHT LINES CUT BY A TRANSVERSAL: Case A: When the two straight lines are not parallel

We have progressed through three levels in our study of lines: 1) the straight line, 2) two lines, and how they are positioned in relationship to each other; that is, convergent, divergent or parallel, and now 3) three straight lines: two that are convergent, divergent or parallel and a third one which crosses both, called the transversal. We consider first that transversal crossing two lines which are not parallel.

Material

1. The wooden board, representing the plane.
2. The box of sticks and the box of geometry supplies.

Presentation

1. From the box of sticks, take two yellow sticks (20 cm.) and fix them (non-parallel) to the board with tacks. (Use a regular sized sheet of paper under the sticks so that it can be identified within certain limits.)

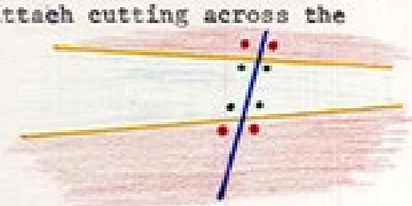
1. We know that these two straight lines are not parallel. And we know that both of them go on to infinity on both sides. This is the plane and both of the straight lines lay on the plane.

2. Color red outside the two lines to note the external and then blue within the two.

CONCEPT: the interior and external* parts.

2. This plane is subdivided by these two straight lines into three parts. There is one part which lies between the two. It is called the interior. There are two parts that lie outside the straight lines---these are the external* parts. I'll color the external* parts red; the internal blue

3. Take another stick (blue---with holes all along it) and attach cutting across the first two.



3. This line which cuts the first two is called the transversal.

- exterior angles identified (#5) (external)
- interior angles (#5)

4. Count the angles formed by placing a colored tack on each one. HERE USE ALL THE SAME COLOR TACKS. . . WE ARE INTERESTED IN ALL THE ANGLES.

4. How many angles are formed when two straight lines are cut by a third one lying on the same plane? We find that there are 8.

5. Establish which are the external and the interior angles, distinguishing them with different colored tacks now---red for external and green for interior.

5. Which are the external* angles and which are the interior angles? We know that the blue area is internal and the red area is external. So we can easily identify the angles. How many external* angles are there? How many interior angles? The four external* angles lie on the external part. The four interior angles are formed in the interior part. Two straight lines cut by a transversal forms four external* and four interior angles.

* Correct nomenclature: exterior
exterior angles

6. Note the etymology of the terms.

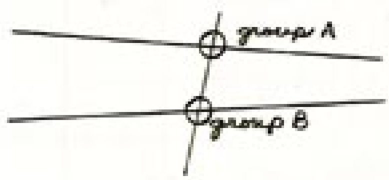
(English usage is "interior," the comparative form: European, Italian usage is "internal," the positive form of the Latin word.)

6. The word "external*" means "outside." Outside what? The straight lines. "interior" (or internal) means inside the straight lines.

7. Give the nomenclature of the angles.

NOTE: alternate, alternating, alternative.

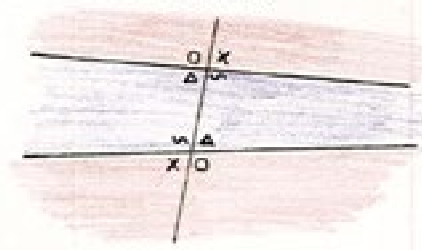
Point out the two groups of angles, an important concept.



7. Alternate is a word meaning one on this side and one on the other side. It means we take one from each side. As if we were walking down the middle aisle and we called one student from one side and another from the other side. From the other side of what? The aisle. The line we are walking. And here we mean on one side or the other of the transversal.
SO ONE SIDE OF THE TRANSVERSAL IS ONE THE RIGHT AND ONE SIDE IS ON THE LEFT.

There is one more specification. When we choose alternate angles, we must choose one angle from this group of two external and two interior angles; and one from this lower group of four angles.
We must compare one angle from one group with one angle from the other group.
If we take two angles belonging to only one group, we might as well only have two lines.

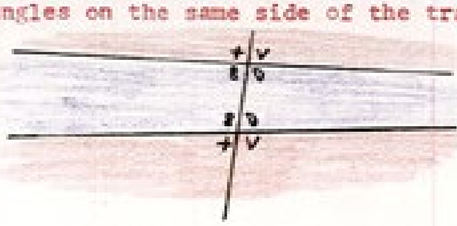
8. Discover with the child the two pairs of alternate external* angles and the two pairs of ~~XXXXXXXX~~ alternate interior angles.* Mark each pair with a different color tack. . .OR the external* and interior with different colors.



* Correct nomenclature: exterior; exterior alternate angles; interior alternate angles

8. So we choose one angle from group A and one from group B.
I choose this angle first in the red external* part.
Now I must find another external* angle but I must look on the other side of the transversal.
Putting both qualities together we have external alternate angles.* (xx)
Is there another couple? (oo)
There are two pairs of external alternate angles.*
Now I choose this angle. I must choose one like it on the other side of the transversal. This angle is on the interior part. Where must my second one be?
These angles are both interior angles and on different sides of the transversal. They are interior alternate angles. (~~)
Can you find another pair? (Δ Δ)

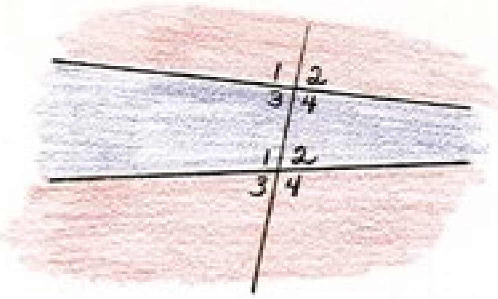
9. Discover the external angles on the same side of the transversal and the interior angles on the same side of the transversal.



Now we are considering only one side of the transversal.
We mark one angle on the external part and then another: we have two external* angles on the same side of the transversal.
Is there another pair? (+ + ∨∨)
Now we consider those angles on the blue part. We find one and then another: on the same side of the transversal.
We have two interior angles on the same side of the transversal. (§§ ŸŸ)

THE STUDY OF LINES. . .
Two Non-Parallel Lines and a Transversal. . .

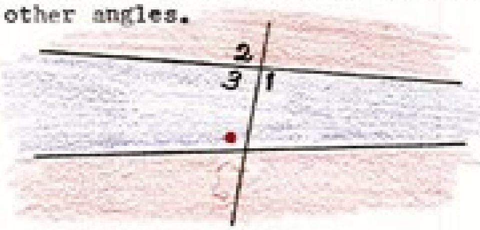
10. Discover the corresponding angles: mark the pairs with different colored tacks as in the numbers 8, 9.



10. Let's choose any angle.
Now let's choose one on the same side, but put first a red and then a blue.
We now have two angles on the same side of the transversal, but one is blue (interior) and one is red (external).
They are corresponding angles.
WHY---with those strange characteristics, are they called corresponding?
Both are on the same side of the transversal. . .
and now both are above the straight line.
How many more pairs of corresponding angles can be discovered?
If I choose this one, what will be the corresponding angle?
There are four pairs.

EXERCISE: Ask the child to represent, draw, all the angles discussed up to this point.

EXERCISE: The child chooses one angle. . . And he discovers its roles in relation with the other angles.



maybe this one with the red tack.
Let's discover all the roles this one angle can play in respect to the others.
With #1, we have interior alternate angles.
With #2, we have corresponding angles.
With #3, we have two interior angles on the same side of the transversal.

11. Present the Classified Nomenclature materials.

12. Grammar analysis of the new phrases: interior alternate angle, exterior alternate angle, external angles on the same side of the transversal, interior angles on the same side of the transversal, corresponding angles. He writes the phrases and matches the corresponding symbols.

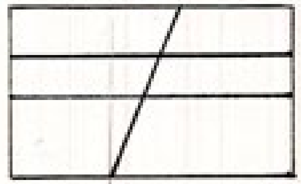
DIRECT AIM: 1) The knowledge of the nomenclature of the angles formed by two straight lines cut by a transversal.
2) Study of plane figures

AGE: after 8½ years

TWO STRAIGHT LINES CUT BY A TRANSVERSAL: Case B: When the two straight lines are parallel

Presentation

1. Prepare on the plane two parallel sticks cut by a transversal.
MATERIAL NOTE: Prepare also four large pieces of paper on which are drawn the parallel lines cut by a transversal in heavy black lines----those lines all running to the edge of the paper.
Introduce these sheets which correspond to those lines on the plane.

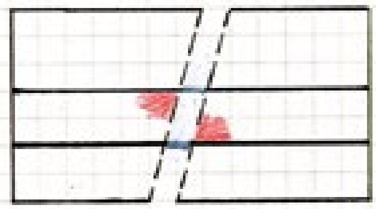


This is a plane, like the board.
The lines go to the end of the paper to show that they go on to infinity at both ends.
What can we say about these two lines?
Notice that the three lines made with the sticks on the board have the same positions.

THE STUDY OF LINES. . .
Two-Parallel Lines Cut by a Transversal. . .
Presentation

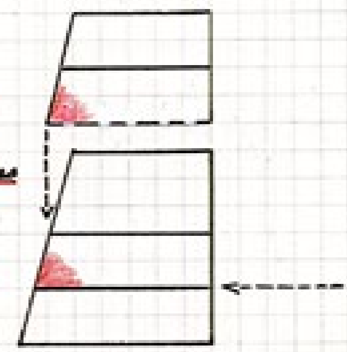
2. Demonstrate that the alternate angles are equal, using the paper sheet.
The child identifies two angles, a pair of alternate interior angles.
On the paper sheet he colors them both in. (the same pair he showed on board)
CUT along the transversal.
FOLD the two halves to show their equality.
CUT SECONDLY along one of the parallels to isolate the one angle on one side.
THEN SLIDE the angle from the top down to meet the other angle. . .or along the second ray of the angle---one of the parallel lines.

2. This is a pair of interior alternate angles.
We have to show that they are equal angles.

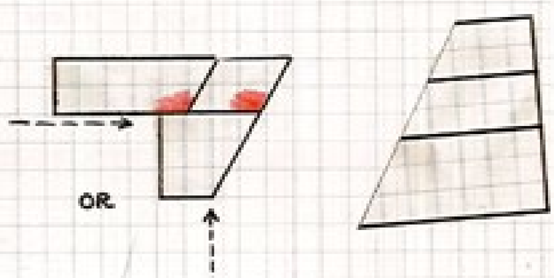


3. With a new piece of paper each time, make this demonstration with each of the four pairs of alternate angles.

Now I can say that this pair of interior alternate angles is equal.



4. Ask the child to prove now that the corresponding angles are equal.
   a) He chooses one of the four pairs of corresponding angles and marks those two with tacks on the plane.
   b) He then uses one of the prepared sheets of paper, and colors in those two angles which he has chosen.
   c) He cuts along the transversal and puts aside the part which is not needed---that part without the angles.
   d) Now he cuts along one of the two parallel lines.
   e) Then he slides the angle from which one side has been cut clear, onto the other angle to prove its equality. He may slide it onto the angle along either ray of the angle. . .from the bottom or from the side.

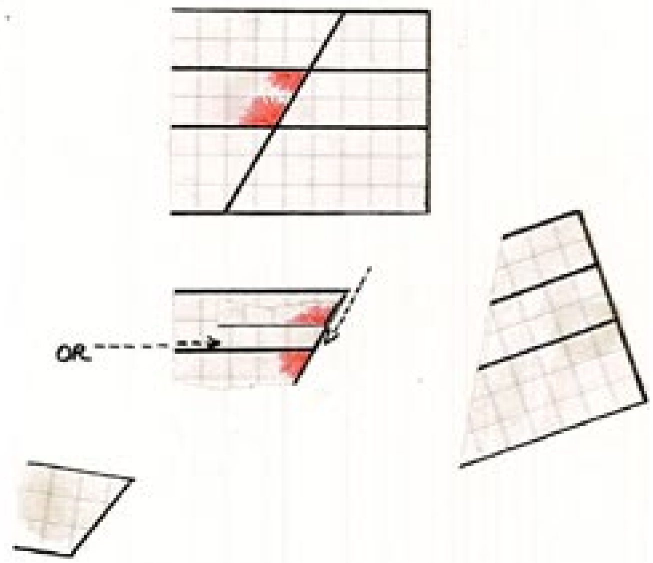


This is an angle we are considering.
It has two sides.
So we can slide the second angle for comparison onto it along either ray. . . from the bottom or from the side.
Now we have seen that pairs of corresponding angles are equal.

5. The child may repeat this exercise with other pairs of corresponding angles.

6. The child now proves that a pair of interior angles on the same side of the transversal are NOT equal, but rather that together they form a straight angle; that is, they are supplementary.
   a) The child chooses a pair of interior angles on the same side of the transversal, marking them with tacks on the plane.
   b) Using the prepared sheet of paper, he then colors in those corresponding angles.
   c) He cuts first on the transversal, discarding the one side that is not necessary.
   d) Then he cuts BETWEEN the parallel lines.
   d) He takes either part and cuts away the small part where the angle is not drawn.
   e) He then takes that part, with the angle cut cleanly on one side, and slides it from the side or from the top until it rests SIDE BY SIDE with the second angle.

6. Now we want to discover the relationship of two interior angles on the same side of the transversal.
We will prove NOT that those two angles are equal, but that together they form a straight angle. . .that they are supplementary.



One angle has two sides.
The other angle has two sides.
Now we don't want to superimpose them, but we want to put them side by side.
The two angles are supplementary because they form a straight angle.

7. The child now proves that a pair of exterior angles on the same side of the transversal are also supplementary and form a straight angle. He proceeds as in the above proof. After making the cut along the transversal he may then:
   a) Cut between the parallel lines as before,
   b) Cut along one of the parallel lines that forms the side of one of the exterior angles,
   c) Fold along one of the parallel lines where the b) cut was made: this last method the poorest.

   Then he slides the cut-away angle from the side or from the top to show that two exterior angles on the same side of the transversal form a straight angle: that they are supplementary.

DIRECT AIM: Knowledge of the theorems

INDIRECT AIM: Preparation for the detailed study of the parallelograms

AGE: 9

NOTE: This chapter is not fundamental. Part A, those proofs made with the straight non-parallel lines cut by a transversal, is more essential than Part B, those with the parallel lines.

THE STUDY OF ANGLES: WHOLE, STRAIGHT, RIGHT, ACUTE, OBTUSE

Material

1. The wooden plane, the box of sticks, the geometry supplies.
2. A sharp red pencil.

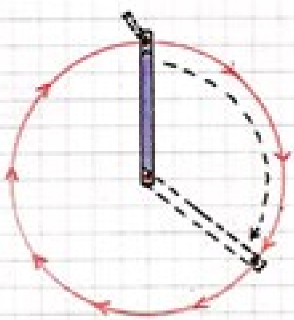
Presentation

1. Take two sticks of different lengths, the longer one having many holes along it. Fix the smaller stick to the plane, on the top end. Then superimpose the second longer stick on top, flush at the bottom end. Take a nail with a red head and fix both together at this bottom end. The head represents the vertex of the angle. Now the bottom stick is fixed and the top one can move.

1. Each of these sticks is a ray. Both start from the same point. Because a ray is very thin, when we fix the sticks this way, it looks as though we had only one ray

2. Put the red pencil point through the last hole of the loose top stick. Move that ray, leaving the trace of the pencil as it travels around to draw a whole angle.

2. This is an angle. . .
this is an angle. . .
this is an angle. . .
this is a whole angle. It is a whole angle because I have gone completely around.



3. Etymology:

3. "Angle" is a word that has several references from different languages. It comes from a word that means "fish-hook." What does a fish-hook look like? Where is the space that might show us the distance between two rays? It also meant "ankle" and "limb" from Sanskrit words. How do our limbs form angles?

4. Repeat the movement of the top stick now, stopping at a straight angle.

4. This is a straight angle because the two rays form a straight line. They are now simply a prolongation of one another.

5. Repeat the movement of the top ray, this time stopping at the right angle. Show the measuring angle as comparison.

5. This is a right angle. And this is a right angle, our measuring angle. We call it the measuring angle because it will be our point of reference, just as the plumb line was; and with it we can define two other important kinds of angles. "Right angle" comes from a Sanskrit phrase which meant "stretching up." Perhaps it referred to the position of one's body when he sat up from a laying position. Would we form a right angle that way?

THE STUDY OF ANGLES: Whole, Straight, Right, Acute, Obtuse. . .
Presentation. . .

6. LEAVE THE MEASURING ANGLE ON THE PLANE. Move the top stick to the position of an acute angle, defining it in terms of the measuring angle. Then show the obtuse.

6. This is an acute angle because it is smaller than the right angle.
"acute" – from "sharp, pointed blade".
And this is an obtuse angle because it is larger than the measuring angle.
"obtuse" – meaning "dull, wide."

7. Show that the straight angle is two measuring angles, flipping the measuring angle over to prove it. Then show that the whole angle is four measuring angles, four right angles. . .flipping it two more times.

7. Now our measuring angle can show us something else very interesting.
We see that the straight angle is equal to two right angles.
And the whole angle is four right angles.

8. Give the nomenclature of the angle.

8. This is the vertex of the angle.
These are the sides of an angle.
This is called the size of the angle.

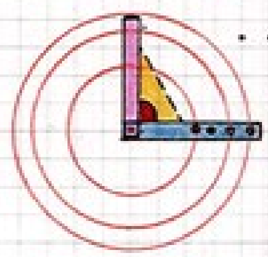
9. REPEAT THE PRESENTATION, having now passed the pencil point two holes down on the top stick. Now the pencil point makes a smaller circle: we are shortening the radius.

THEOREM: The size of the angle doesn't depend on the length of the sides.

9. This is an angle. . .
this is an angle. . .
this is a whole angle.
This is an angle. . .this is a right angle.
this is a straight angle. . .this is
an acute angle. . .an obtuse angle.
Let's measure the size of the right angle and the straight angle.
The sides became shorter, but the size of of the angle is the same.

10. Move the pencil point down two holes again. . . . . .repeat.

11. Introduce nomenclature materials.



A GAME: Rotating the top ray, ask the children to give the name of the angle.

acute. . .acute. . .acute. . (many times)
right angle. (one time)
obtuse. . .obtuse. . .obtuse. .(many times)
straight angle. (one time)
larger than the straight angle. . .larger than the straight angle. . .(many times)
whole angle. (one time)

NOTE: A Mathematicians Dispute: Whether the right angle is the intersection between the set of acute angles and the set of obtuse angles OR if it is the limit of the set of right angles and that of the obtuse.

AN EXERCISE: (from the commands) The Measuring of the Angles: child takes the series of plane figures from the cabinet. He traces the inset of the frame, colors the figure and maybe cuts it out. Then he takes the measuring angle and identifies each angle, writing: right, acute, obtuse. . . He does the same work with all the figures of the cabinet. When he comes to figures limited by curved lines he writes "I HAVE NOT BEEN ABLE TO MEASURE THE ANGLE." or "I DON'T KNOW." This work is good and especially interesting with the polygons. We are preparing for:

THEOREM: The smaller the number of sides in the figure, the smaller the size of the angles. Thus, when I come to a circle, with infinite sides, I can't even measure the angle.

THE STUDY OF ANGLES. . .

ANGLES: ADJACENT, VERTICAL, COMPLEMENTARY AND SUPPLEMENTARY ANGLES

Material
1. The plane, the box of sticks, the geometry supplies

Presentation: First Case of Adjacent Angles (Consecutive)

1. Ask the child to take four sticks from the box, two of equal length and two others. Then have him construct two angles, each angle having as one side one of the pair of equal-lengthed sticks. (CONSTRUCTION: One angle should be constructed with the paired stick below, and one with it on top)

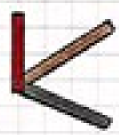
2. Repeat the nomenclature of the angle.

2. This is the vertex.
Show me the sides.
What is this? the size of the angle.

3. Put the two angles together on the plane, superimposing the two equal sticks.

3. Because these two sticks are two rays, they are very thin.
Together they look like one.

4. Now take the equal stick off the top of the one angle, place the other side of that angle below the other two angle sticks and fix them together with one paper fastener so that the equal stick is IN THE MIDDLE.
Give the name "adjacent."



4. In fact, it is enough to use only one stick to show both these rays.
Now these angles are very special:
they have a common vertex and one common side.
They have a special name: adjacent angles.
The word adjacent means "one close to another."

5. Ask the child to construct another pair of adjacent angles. Three-period lesson, including new nomenclature.

5. What are these angles?
Show me the common vertex.
Show me the common side.

6. Remove all but one set of adjacent angles. Fix two sides, including the common side and leave the other one free. Move it and ask:

6. What are these angles?
Now? Now?
Why are these angles adjacent?

ACTIVITIES: The child looks for examples of adjacent angles in the environment.
Command: the child draws adjacent angles, writes the name, and then writes a definition in his own words.

Presentation: Another Case of Adjacent Angles

1. Ask the child to construct adjacent angles. Fix the first and second (common) sides to the plane, leaving the third side free.

1. How many angles are here?
What are these angles?

2. Move the third stick to the position of a straight angle.

2. Here is a special kind of adjacent angles.
Which is the common side?
We can see that the other two sides are prolengations of each other.
What kind of an angle do they form?

3. Exchange the two non-common sides for one longer stick.

3. Now we can change these two non-common sides for one stick, and we still have adjacent angles.

4. Show both the adjacent angles constructed in the first presentation and the special case here constructed; three-period lesson. Show that the first can be the same as the second.

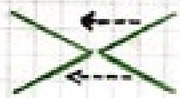
5. Introduce the Classified Nomenclature materials.

ACTIVITIES: The child looks for examples of this special case of adjacent angles in the environment.
The child draws adjacent angles in several different positions, labels them and writes a definition.

Presentation: VERTICAL ANGLES

1. Ask the child to take four equal sticks and unite them, in pairs; forming two angles.

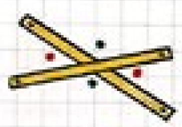
2. Show both angles on the plane. Then, keeping one pair still, with vertex turned towards the center of the plane, move the other angle until the vertices join.



3. Dispense with one paper fastener, fixing all four sticks at the common vertex with one. Note that the two opposite sticks are prolongations of each other

3. Now the two vertices of the two angles coincide to form one.
We can fasten all four rays together at this common vertex.
And we see that this side of one angle is the prolongation of this side of the second angle.

4. Point out the vertical angles.
SUBSTITUTE TWO LONG STICKS for the four shown.



4. With these two angles, we have formed four angles.
The angles which are opposite of each other are called vertical angles.
Let's mark one angle.
Which angle is opposite it?
Then these two angles in front of each other are vertical angles.
Is there another pair?
With these two lines, each formed of two sides of the angles we began with, we have formed two pairs of vertical angles.

Second Presentation: Vertical Angles Are Equal Angles: Sensorial Proof

Material
1. Prepared rectangular paper on which are drawn vertical angles, those rays extending to the very edge of the paper.
2. The plane, sticks and geometry supplies.

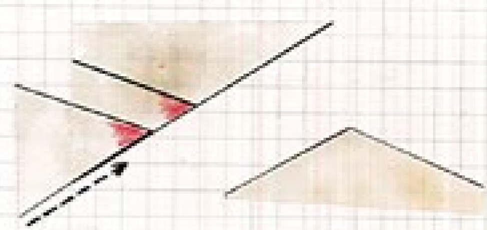
Presentation

1. Ask the child to show vertical angles on the plane, using two longer sticks.

1. Which are our vertical angles?
Let's mark one pair with green tacks and the other pair with red tacks.

2. The child chooses one pair of vertical angles, takes one of the prepared sheets and colors in the corresponding chosen angles. Then:
   a) he cuts along one line
   b) he takes one piece and cuts away along one side of the angle
   c) he slides that clear angle from the bottom along the side of the other vertical angle,
   and shows that they are equal.

2. We want to show that vertical angles are equal.

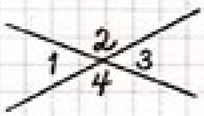


3. Repeat with the other pair of angles.

Angles: Adjacent, Vertical, Supplementary, Complementary

Presentation #2: Vertical Angles Are Equal: Abstract Proof

When the child has had experience in measuring angles, we show him the written proof for the equality of vertical angles.



We know that angles 1 and 2 form a straight angle; that they are therefore supplementary and together equal 180°.

So: $\angle 1 + \angle 2 = 180°$

And we know that this is also true of angles 2 and 3.

So: $\angle 2 + \angle 3 = 180°$

And: we have a common element in the two statements: $\angle 2$.

Then $180° - \angle 2 = \angle 1$
and $180° - \angle 2 = \angle 3$
so $\angle 1 = \angle 3$

Age: 9 years

Another Proof:

$$\begin{aligned} \angle 1 + \angle 2 &= 180° \\ \angle 1 + \angle 4 &= 180° \\ \angle 2 &= \angle 2 \end{aligned} \quad \text{or} \quad \begin{aligned} \angle 2 + \angle 3 &= 180° \\ \angle 2 + \angle 1 &= 180° \\ \angle 1 &= \angle 3 \end{aligned}$$

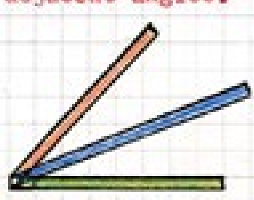
Presentation #3: COMPLEMENTARY AND SUPPLEMENTARY ANGLES

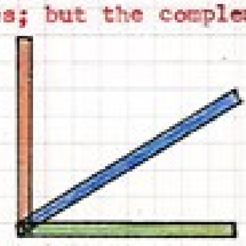
1. Ask the child to take four sticks and construct with them two angles. Recalling the work with adjacent angles, bring one angle to the other and ask the child to construct adjacent angles, showing one common side for the two internal sides of the two angles.

2. Invite the child to form a right angle, using the measuring angle as a guide, between the two external (non-common) sides of the two angles.

2. When two angles together form a right angle, they are called complementary angles.
"Complementary" means that one angle "completes" the other to make a right angle.
The goal of these two angles is to become an angle like the measuring angle---a right angle. But they cannot do it alone---they need each other.

3. Ask the child to construct another set of adjacent angles. Then have him transform those adjacent angles into complementary angles. NOTE: adjacent angles are not always complementary angles; but the complementary angles are adjacent angles.





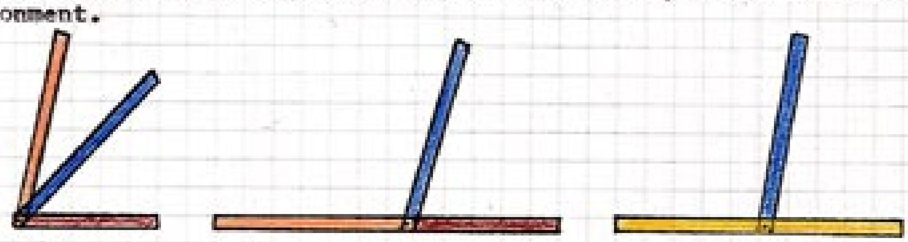
ANGLES: ADJACENT, VERTICAL, SUPPLEMENTARY, COMPLEMENTARY. . .
Presentation: Complementary and Supplementary. . .

4. Ask the child to again take four sticks and construct two angles. Then eliminate one of the internal sides when the angles are moved together, thus forming a common side.

5. Now move one of the non-common sides to the position of prolongation with the other non-common side, showing a straight angle. Give the nomenclature.

3. With these two angles, we have formed a straight angle. These angles are called supplementary angles. They are also adjacent angles. Two angles which together form a straight angle and are adjacent are called adjacent supplementary angles.

6. Construct another pair of angles, show them as adjacent straight angles. Then substitute one stick for the two non-common side sticks which are now a prolongation of each other. NOTE that we still have the two angles, that they are still adjacent supplementary angles. Look for such examples of this arrangement in the environment.



A CLARIFICATION: Adjacent angles can be complementary; but complementary angles DO NOT HAVE TO BE adjacent.
Two adjacent angles are supplementary angles IF and ONLY IF the sides which are not common form a straight line.

CLASSIFIED NOMENCLATURE: THE STUDY OF ANGLES. . .

MEASURING ANGLES

Material

1. The metal insets---circles, triangles, squares.
2. The geometry cabinet, all drawers.
3. The two series of geometrical figures on cards: surface and contour.
4. The measuring angle.
5. The Montessori protractor.
6. A regular protractor: round, 10 cm. diameter
7. A stick picked at random from the box of sticks.
8. Paper, pencil, red pencil.
9. A compass.

Presentation

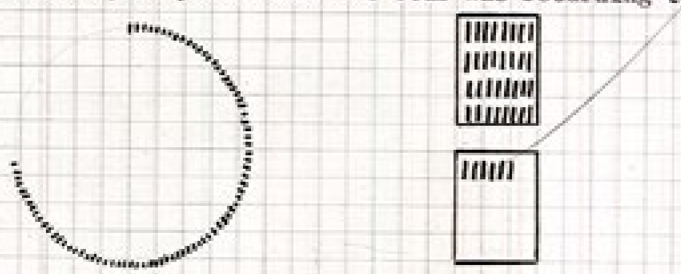
1. Take the 1/3 circle metal inset; propose to measure the size of the angle. Try a ruler, the measuring tool with which the child is familiar.

1. Today I want to measure the size of this angle.
With my ruler, I can measure the side here of this angle, BUT I want to measure the SIZE.
It's no good.
The ruler is only good for measuring straight lines.
We need a special instrument;
I am going to give you a circle to measure the size of that angle.

2. An introduction to the protractor:

The study of astronomy was begun by the Sumerians. Astronomy is the study of our solar system. (The Sumerian civilization runs parallel with and before that of the Babylonians---often the Sumerians, Assyrians and Babylonians are all referred to as the Babylonians.) The Sumerians were a well-organized people. They kept track of their clothes, their food and many other things. The priests did this. They were clever, too, in land measurement and division, using geometry and arithmetic calculations. They even used the circle. They did not write words, but used pictures on clay tablets.

It was also the priest's duty to keep track of the planets. They were especially interested in the sun, the moon and five of the planets. They gave those planets they observed the names of their weekdays: Sunday was the sun's day, Monday the day of the moon, Tuesday the day named after Mars, Wednesday was named for Mercury, Thursday for Jupiter, Friday for Venus and Saturday for Saturn. As the priests studied the sky and the solar system, they were soon able to recognize the position of the stars. At a certain point they decided to take one star and watch it. We believe that the star they chose was actually a planet, or it may have been the dog-star, Cirius. They marked the position of that star on the first day; and on one of the clay tablets, they made a mark to show one. Then they put a mark for each day and noted the position of the star again. For many days they marked the place where they saw the star in the sky, keeping an exact record of how many days passed. And at a certain point, they saw that the star was returning to the point where it had started.



When the star marked the exact point where it had begun, they were finished. And they had 360 points and 360 lines marking the days of those points. They concluded that the star must have gone in a circle. And that journey of 360 days became the Sumerian year. The word circle comes from a word meaning "period of time."

MEASURING ANGLES. . .
Presentation. . .

The Sumerians divided that circle that the star had tracked into 360 sections, each one representing one of their days. And that is now called the degree of our circle. Our circle has 360 degrees. This little degree has a symbol: a tiny circle (°), symbolizing the star's walk around the sky. The word degree comes from a Latin word "de gradus" meaning step.

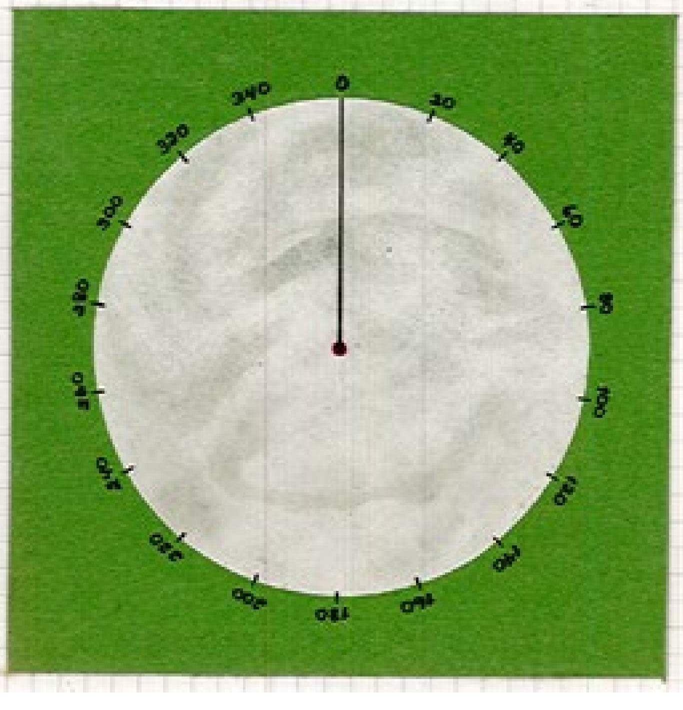
Here is an angle.
When we measure the size of an angle, we are measuring the path of that star. What distance it's movement from this top line to the lower one is.
And we name that distance in degrees.
We write the number and then the symbol for degrees.

3. Show the metal circle whole inset, removing the inset and tracing within the frame.

3. Imagine that great circle in the sky, becoming smaller and smaller and smaller, until it is a circle just this small.
If we go around this frame, all the way, how many steps would the star have taken?
How many degrees would we have counted?

4. Introduce the Montessori protractor and examine it with the child. First fit the circle inset within it, noting the exactness with which it fits. Give the etymology of the instrument.

4. This is the Montessori protractor.
On it those 360 degrees are marked.
Let's see if this circle is the same as our whole circle.
It is exactly a whole circle.
But around this circle there are numbers which show the degrees.
The instrument is called a protractor.
Protractor is a Latin word which means "to substitute for," "to trace."
Let's trace our finger around its frame and read the degrees.
When we come back to 0°, we have counted 360°, so we would call 0° also 360°.
The red dot in the center is called the vertex. (Vertex comes from the Latin meaning "highest peak" or "to turn.")
The black line is called side.



MEASURING ANGLES. . .
Presentation. . .

5. Show the measuring angle on the protractor, demonstrating that we can measure four right angles; flipping it over as in the angle presentation.

5. How many measuring angles are there on the protractor?
With this measuring angle, we could tell whether our angle was a right angle, an acute angle or an obtuse angle. . .
so we could call this our first protractor.
With our new protractor, we can measure our angles with great precision, measuring them exactly according to their degrees.

6. Measure on the protractor the angles of the circle insets. We begin measuring these fractional insets because each has only one angle; and the element is therefore isolated.

We measure the insets in four groups:

- #1: 1/3, 1/6, 1/9 (all these angles showing a number of degrees which is actually written on the protractor.)
- #2: ¼, 1/5, 1/8, 1/10 (these measuring to an even degree, but one not written on the protractor)
- #3: ½, 1 (special problem of no vertex)
- #4: 1/7 (another problem--- an irrational number)

6. What is this? 1/3 of the circle.
Let's look at this angle.
Show me the vertex of the angle.
Show me the sides.
Show me the size of the angle.
That is what we want to measure---the size
To measure the size of an angle on the protractor, I place the vertex on the vertex; and one side of the angle on the black line of the protractor---side.
Then I look to see where the other side falls on the protractor.
And I read the number of degrees which tells me the size of the angle.
When we measure 1/4, the number of degrees is not written on the protractor.
How will we read the degrees?
½ has no vertex---where would the vertex be? In the middle.
But we cannot see the vertex because the sides are prolongations of each other.
So we place one side on the black line--- what does it read? 180°.
And the whole has no vertex. . .neither does it have sides.
It measures the whole circle. . .360°.
Now we want to measure 1/7---
I can't tell you exactly what it measures. . .51. . .and. . .
But, though we try to divide 360 by 7, we will never come out with an exact quotient. It represents an irrational number.
So we say that this angle measures approximately 51°.

7. Conclude that now we have two ways to measure any angle: we can use the measuring angle to tell what kind of an angle it is; and we can measure the exact number of degrees by using the protractor.

8. Note that any angle can be measured with the protractor. Try the ½ square cut on the diagonal. And several other examples from the insets.

8. We can measure all of the insets with this protractor.
Here we have a fractional inset with three angles. We can measure each one---
We begin with one angle, placing vertex to vertex and side to sin. . .then read the size of the angle in degrees.

9. Measure the 1/16, using a stick to prolong the side where the degree is read.

9. This angle has very short sides.
In order to determine the size of the angle, we must take a stick and prolong the second side so that we can read the number of degrees.

ACTIVITIES: The child can draw the inset figure, color it and cut it out, then measure its angles and paste it in his notebook, noting the number of degrees.

Special Commands: a) Measure the fraction insets of the circles from 1 - 1/10 and write the number of degrees in each angle.

b) Make a list of the triangle fractions and the number of angles that each one has. Then add to your list the number of degrees in each angle, measuring them on the protractor.

AGE: 6½ - 7

Presentation #2: Introducing the regular protractor

Material: In addition to the materials used in the first presentation, use here a regular protractor with 10 cm. diameter.

1. Show both the Montessori protractor and the regular circle protractor.	1. This is another kind of protractor. Where is the vertex on this one? Where is zero? And where is the black line sin? On this protractor it is indicated by two smaller lines that cross the vertex.
2. Show that the new protractor fits exactly into the Montessori protractor.	2. It is interesting that our new protractor fits exactly into the circle of the one we have been using. With this protractor, we can measure the same figures; and we may find that we can more easily measure other angles of figures, too.
3. The child can measure the first card series for variety here---and also good exercise in using the protractor on a flat surface.	3. Let's read the numbers on this protractor. Now we have the degrees written every 10°. We measure the size of an angle with this protractor just as we did with the first. Vertex to vertex, side to line, and we read the degrees at the other side. We always make sure that our first side coincides with zero.

NOTE: The child can now measure any figures enclosed by straight lines.

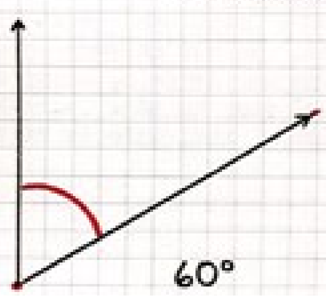
ACTIVITIES: The child practices with the new protractor by measuring the angles of the figures in the geometry cabinet.

He can draw geometrical figures, color, cut and measure them, then pasting them in his notebook and writing the size of the angle in degrees.

Use interesting Commands: a) Using a prepared angle, the child measures that angle and writes the degrees he measures.

OR

b) THE INVERSE. The child draws an angle of a specific number of degrees: (1) first he marks the vertex in red and makes one ray as an arrow from that vertex, (2) using the protractor from the drawn side, he measures the number of degrees, marks it and then draws the line from the vertex to the mark. (3) He shows the size of the angle in red and writes the degrees.



MEASURING ANGLES. . .

THE OPERATIONS WITH THE PROTRACTOR

Material

1. The Montessori protractor.
2. The metal insets. . .circles.
3. A compass

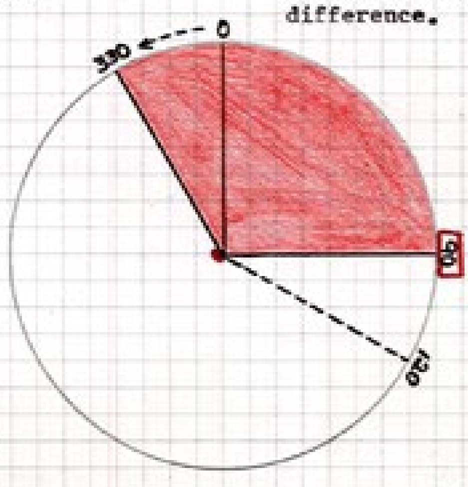
Presentation: Addition, Subtraction, Multiplication

1. Addition: $60^\circ + 90^\circ =$ That is a 60° angle plus a 90° angle. So first the child must choose those two insets which measure 60° and 90°. He may have remembered which fractions those are or have them written in his notebook; he may have to discover the sixth and the fourth fractions. THEN he first places the 1/6 to show 60°, placing vertex to vertex and side to black line. Then he places the 1/4 to show the 90° angle immediately after the first, side to side. He reads the total degrees on the second side of the second addent (90°). And he writes: $60^\circ\angle + 90^\circ\angle = 150^\circ\angle$

   NOTE: The child can add any fractions from 1 to 1/12 now without dealing with common denominators. He simply lays the two angles side by side and reads the total degrees.

2. Subtraction: first method. $120^\circ\angle - 30^\circ\angle =$
   The child shows the angle on the protractor, then rotates it backwards past zero the number of degrees of the subtrahend. He now reads the new position of the second side of the original angle for the difference.

2. I have 120°.
   I have to take away 30°.
   So I first show my 120° angle on the protractor, vertex to vertex, side to sin. Then I push the inset fraction backwards 30° behind zero.
   Now I read the degrees shown where the second side falls, and that is the difference.



$120^\circ\angle - 30^\circ\angle = 90^\circ\angle$

Subtraction: Second method. This time, as in addition, the child places first the minuend angle on the protractor and then, side to side, the subtrahend angle. Then he moves both angles together, rotating them backwards past zero until the second side of the subtrahend angle reads the degrees of the minuend angle. . .that point where the second side of the minuend angle originally marked. He removes the subtrahend angle and reads the difference where the second side of the minuend now lies. Using the problem above, then, when he places the fractions together on the protractor, together the degrees shown will be 150°. He rotates them together backwards until the 1/12 second side shows 120°. He removes that second fraction and reads the difference at the second side of the 1/3 which will be 90°.

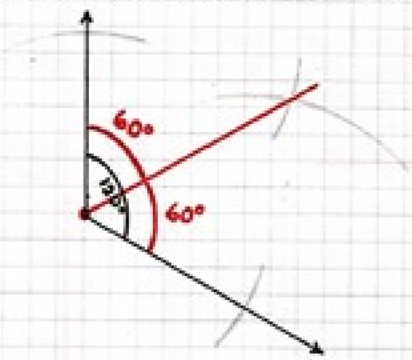
3. Multiplication: $45^\circ\angle \times 3 =$ Here the child shows on the protractor the same fractional inset (1/8) three times, lying side by side, the first one beginning at the black line, vertex to vertex, side to sin. He reads the number of degrees marked by the second side of the last fraction's angle placed down. So: $45^\circ\angle \times 3 = 135^\circ\angle$

Presentation: Division of An Angle: Bisecting the Angle

1. Introduce the compass and show the child on paper how to use it to bisect an angle.

1. The compass is an instrument we can use in our work with angles and in the drawing of circles. It is a tool of many uses. The word compass means "to measure," or "to go around."

We can divide this angle exactly in half with the compass. We begin by drawing an angle. This is an angle of 120°. Then we put the point of the compass at the vertex, and we show arcs on each of the two rays to make an exactly equal measurement on them. From those two points where the arcs cross the rays, we use the compass point and draw arcs within the size of the angle. Then we join the vertex to the intersection of those arcs. And we have bisected the angle---we have divided it in half.



2. With the protractor the child measures the newly formed angles, checking the division: $120° \div 2 = 60° \angle$.

3. Introduce the Classified Nomenclature: C8a on the Bisector of an Angle.

ACTIVITY: The child does the operations in his notebook, drawing the fractions and writing the operations.

AGE: 7½

Presentation #3: Other Protractors

Material
1. The Montessori protractor.
2. The 10 cm. diameter protractor.
3. A round protractor with a slightly larger diameter and another one with a much greater diameter.
4. Two 180° protractors.

Presentation

1. Present the Montessori protractor, the one the child has been using, and the two larger round ones.

1. Here we have four protractors. This is the Montessori protractor, the first one we used. This 360° protractor has a diameter of 10 centimeters. This round protractor still shows the 360°, but it has a bigger diameter. And this round protractor has a much bigger diameter. Only one of these protractors will fit in the Montessori protractor; but each one shows 360°.

MEASURING ANGLES. . .
Presentation: Other Protractors. . .

2. Measure several fractions, each time measuring the angle with each of the four protractors.

In measuring the fraction on the biggest protractor, the child must prolong the second stick to read the degrees.

2. Let's measure this 1/3 on each of our protractors.
Let's begin with the Montessori protractor.
What is the size of the angle? 120°.
And on this one? 120°
And on the other two. . .the same.
We need our stick to prolong the side of our angle when we use this large protractor.

NOTE: An excellent presentation for the THEOREM: The size of the angle does not depend on the length of its sides.

3. Conclude that our angle will measure the same on any protractor.

3. We could have a very large protractor and this would still measure 120°.

4. Show the two 180° protractors.

4. We can also measure angles with a protractor of 180°.
Where is the vertex on this protractor?
Where do we find sin?
How will we measure an angle?

CONVEX AND REFLEX ANGLES: A presentation made immediately after the measuring of angles.

Material
1. The plane, the box of sticks, the geometry supplies.

Presentation

1. Take two sticks, the longest one with many holes. Fix the short stick to the plane. . .the long one imposed on the first and fixed only at the lower vertex end with the short stick using the red tack. Repeat the presentation of the angle, using the red pencil to draw a whole angle, the child naming the angle as it is drawn.

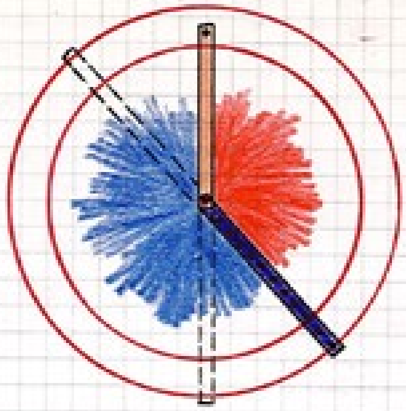
1. acute angle. . .acute angle. . .acute.
RIGHT ANGLE
obtuse angle. . .obtuse angle. . .obtuse.
STRAIGHT ANGLE
greater than a straight angle. . .greater than a straight angle. . .greater than. .
WHOLE ANGLE

2. Move the red pencil down two holes and repeat the exercise, this time the child giving the old name and the teacher giving the names: convex, straight, reflex, whole. . .at the same time.

2. acute angle. . . . convex angle
acute angle. . . . convex angle
right angle. . . .convex angle
obtuse angle. . . .convex angle
obtuse angle. . . .convex angle
STRAIGHT ANGLE
greater than a straight angle. . .
reflex angle
greater than a straight angle. . .
reflex angle
WHOLE ANGLE

3. Define convex and reflex angles.

3. What have we discovered?
The acute angle, the right angle and the obtuse angle are convex angles.
The angles greater than the straight angle is a reflex angle.
The straight angle is neither one.
So we can say that a convex angle is less than a straight angle; and all those angles larger than a straight angle are reflex angles.
The whole angle is the whole angle.



4. Form an obtuse angle and fix it with a tack. Color in the convex angle one color and the reflex another.

4. This is an obtuse angle and it is a convex angle.
This is a reflex angle.

5. Note the classification or the two according to the size of the angle.

5. We can classify these angles according to the size.
The convex angle is not more than 180°.
The reflex angle is more than 180° and less than a whole.

6. We can also classify the angles according to the prolongations of their sides on the ends of the vertex. Show first the sides prolonged in the direction of the rays' extension. Then transfer the prolongations to the sides prolonged on the vertex.

6. We can also classify these two kinds of angles according to the prolongations of their sides of the ends of the vertex.
Let's look at the convex angle:
Show me the sides of the angle.
Show me the size of the angle.
Show me the vertex.
Now let's name the parts of this angle.
What is this?
We can prolong the sides of these angles in either direction. . .but if we prolong them on the outer end, this obviously does not prolong the vertex.
We prolong the sides from the vertex.

7. Analyze the new characteristics of the angles and give a complete definition based on both classifications.

7. Now we still have the two angles.
One is red and the other blue.
What are the new characteristics of these angles?
The red angle, the convex angle, doesn't contain the prolongations of the sides; but the blue one does.
Then, we can define the angles more completely by saying:
The convex angle measures less than 180° and doesn't contain the prolongations of its sides.
The reflex angle measures more than 180° and less than the whole; and contains the prolongations of its sides.

8. Etymology and further analysis of the two angles.

8. The word convex means "vaulted" or "arched." It also comes from a word meaning "to sway" or "to stagger."
Look at the sides of the convex angle.
Can you see the lines that might form the archway above a door?

CONVEX AND REFLEX ANGLES. . .
Presentation. . .

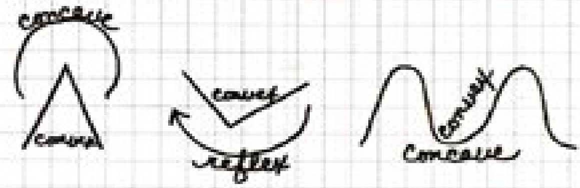
8. . .Etymology and further analysis.

8. The word reflex comes from the word "reflect" which means "to give back," as in "to reflect the image on a mirror." This reflex angle exists as a result of the convex. . .it is a reflection of the other one.
It occupies the space left empty by the other one.
The convex angle here is shown by our sticks of wood and this reflex angle is the empty space remaining.
But without the sticks of wood, the convex angle would not exist.

9. Show a tablespoon and look at it from above and below. Note that it is both convex and concave, giving here the second name for reflex.

NOTE: This is the idea of mountain and valley. The hollow of the valley is the convex angle, in spite of the fact that it is empty and that concave means "hollow." But it is the intrusion of the angle between mountains that we are observing---and the mountains themselves form what is left over---that is the concave configuration.

9. The surface of this spoon can be both convex and reflex at the same time.
But when we are talking about the surface of something, we use another word for the reflex angle: we call it concave.
So now let's observe this spoon turned over.
We must establish whether it is concave or convex.
I see that if I look at it from below, it is convex and from above it is concave.



Presentation #2: The Concave Polygon

Material

1. The constructive triangles, first series, second box. (blue)

Presentation

1. Ask the child to form all the figures he has made previously with this series of triangles. He begins with the original figures: the rhombus, the rectangle, the parallelogram, the square, the trapezoid.

2. Then he shows from that group: the rhombus, the long triangle, the tall triangle, the deltoid, the isosceles triangle and finally the concave quadrilateral.

3. With these variations shown on the mat, name the figures. And discuss the concave quadrilateral.

3. What was the name we gave to this last figure?
Now we can say why it is concave.
The first three figures are convex quadrilaterals. . .this is concave.
We have an obtuse triangle, an acute triangle, and a right-angled triangle.
All of their internal angles are convex.
This is a concave quadrilateral because it has one concave angle.
A quadrilateral polygon is a polygon which has at least one internal reflex angle.

CONVEX AND REFLEX ANGLES. . .

A NEW DEFINITION OF ANGLE

The child has, to this point, defined an angle as something pointed; the reflex angle was not an angle to him. Now we can complete the definition of angle.

The first definition: The part of the plane included between two rays that have the same origin. Now we can say that the angle is each of the two parts of the plane into which it has been divided by two rays having the same origin.

AGE: after $8\frac{1}{2}$

THE STUDY OF THE NOMENCLATURE. . .

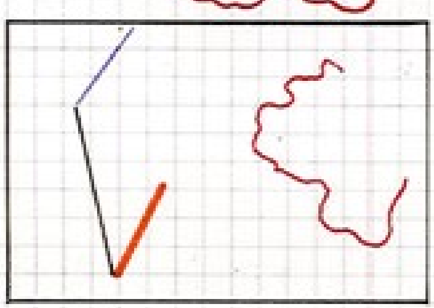
THE FORMATION OF THE REGIONS

Material: the same as in the study of lines
with
a) a red string
b) the last drawer from the geometry cabinet.

Presentation

1. Take the red string between your two hands, holding it to show both the end points.	1. Remember---this was a line segment. This is one end - point. This is the second end - point. This is called the extremity and this is the other extremity. These are the two extremities of the line segment.
2. Place the string casually, loosely curved on the plane.	2. Now this is no longer a straight line segment. It is a curve.
3. Take three of the sticks (of different colors) and fasten them with paper fasteners at two places, leaving the sticks open, unjoined at one place to show a broken line.	3. We have three line segments. One is NOT the prolongation of the other. This is a broken line.
4. Demonstrate the broken line as a combination of segments by bending it.	4. It is as if I had one segment of a straight line and I broke it.
5. Now, on the plane, place the broken line of three sticks along side the string. CONCEPT: of an open line. Important for work with sets.	5. Now let's see if this curve is open or closed. I can go from the outside to the inside. It is open. Let's see if the broken line is open---yes.
6. Now take the string and join the end points on the plane. Close the broken line. CONCEPT: the curve, when closed, forms the simple closed curved region. The broken straight line, when closed, forms the polygon. NOTE: We will also examine those curved regions which are not simple. . .here, bisected.	6. How is this curve now? This is the exterior, the exterior, exterior---I cannot go into the internal part. AND I can no longer go into the internal part here. When a curve or a broken line is closed, it determines two big groups of figures: SIMPLE CLOSED CURVED REGIONS and POLYGONS. When we closed the curve, we had a simple closed curved region. When we close a broken line, we obtain a figure called a polygon.

#5



#6