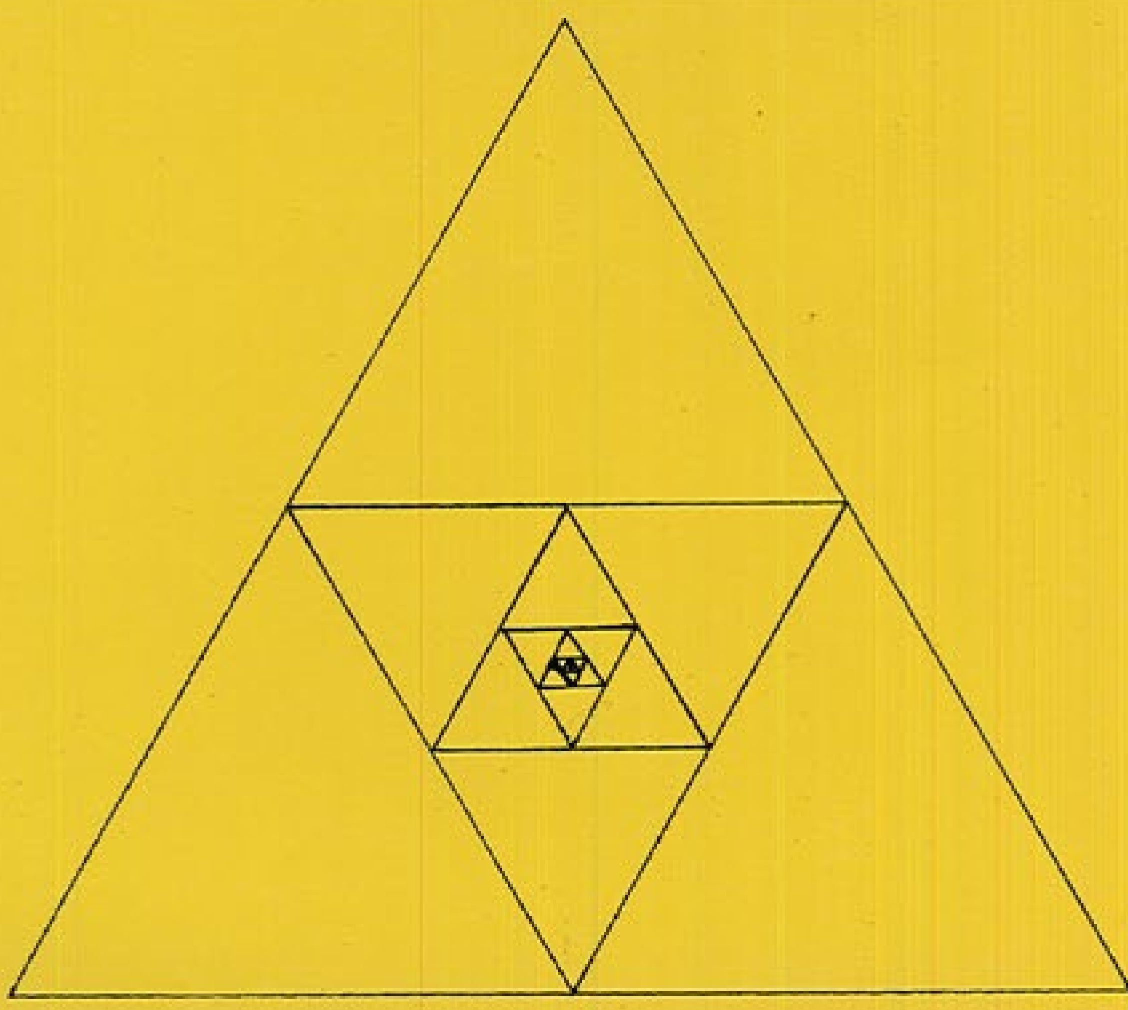
THE CONCEPT OF INFINITY

When the small child builds the PINK TOWER and stands above it to observe the construction from the top, he has an early impression of infinity. It is the constant sequence of cubes that provide the visual concept, the larger cube at the bottom showing the limit and the tiny one at the top moving towards the infinite. That same tower can also be constructed as one would draw squares inscribed, one after the other, within the midpoints (see figure.) The first is the purer construction and the clearest. Both provide an indirect preparation for the concept of infinity with which he will work in the elementary years.

The elementary child is introduced to the idea of a line going to infinity at both ends in the geometry presentation in which we show the string stretching off two spools, that string increasing in length so that it appears to go on and on.

A Picture of Infinity Within a Limit is presented with the charts of the square series numbers 17, 18 and 19. With the fraction metal insets, the child can reproduce, in part, these designs. He may use the square whole, and the square divided into fourths by the diagonals to begin the design of #17. He may also show the beginning of the triangle inscribed within the triangle---see figure---by taking the triangle whole inset from its frame and showing one of the triangle fourths within it. But, having been introduced to the designs on the charts, he may draw similar patterns, beginning with the square, the triangle, or one of the regular polygons. His work is simply to join the midpoints of the figure to form another within it. . .over and over again. . .until he cannot continue because the size is too small. We may explain to the child that with very delicate instruments, this design could go on still further---and then we must let the child use his imagination.



It is important to know, however, that we CANNOT represent infinty graphically because at some point we must stop. . .and infinity has no end. The more we try to give concrete evidence of an abstract concept, the deeper the mistake. We must give the idea and that is enough, knowing that the power of the human mind can imagine the concept of something without end. We must accept that potential of the mind.

Dott.sa Montessori asked why it is so difficult to give the concept of infinity. If we look, not to geometry, but to the ancient philosophies, we find they understood the finite as the perfect, the goodness; and the infinite the imperfect, the evil. For the Greeks the finite was something positive; the infinite negative. As man changed, the philosophers of the romantic period completely changed that Greek concept. The finite became the negative concept and the infinite the only perfect, positive. Then Montessori notes the existential philosophy in which the finite is again goodness--- those individual experiences which cannot be repeated. And the infinite is bad.

For the child, who repeats the primitive schemes of man, it is evident that the finite seems to him good and it is easy to understand; the infinite is difficult. His motivation is limited by his age. But children, just as primitives, have difficulty accepting the infinite.

We may help him begin to identify the concept with the word through a language lesson:

We define the terms line segment as that part of the line having two end points.
ray as having one end point and going to infinity on the other.
line as having no end.

Then, using the first and third terms we may say of the opposite ideas that:
The line segment is a finite line.
The line is an infinite line.

The child matches the symbols to the words.

Now we present an exercise in which we see that we can make abstract nouns from the adjective. "I can ask you to bring me a LONG pencil, but I cannot ask you to bring me the LENGTH."
Length is a noun, but it is something that we cannot see nor bring.

So we may show how a variety of adjectives plus a suffix become abstract nouns, our point here being the abstract concept of finiteness and infinity.

Adjective + Suffix	Abstract Noun
clean + liness	cleanliness
cold + ness	coldness
warm + th	warmth
bright + ness	brightness
finite + ness	finiteness
infinite + y	infinity

Another interesting application to the language work begins with an introduction of the two stories: "The Flying Dutchman" and "The Wandering Jew." The first is an 18th century Norwegian tale at the close of which the sea captain is sentenced to sail around the Cape of Good Hope forever. "You will sail forever." (Heine) In the second story of Ahasuer, we have the tradition of infinite nomadism of the Jews and the command "You will walk forever." Both can be used to demonstrate the intransitive verb. There is no object to receive the action of the verb, and there is therefore no end implied. With the language sentence construction materials, we might show this by placing an arrow directed towards the subject from the verb and another that goes from the verb straight up. Thus in these two phrases we have the image of the intransitive verb. And we also demonstrate infinity as that action which is perpetual, having no end.

In our presentation of Christ: the Center of Man's History we utilize two strips of cardboard with frayed edges to represent time that goes infinitely into the past and the future. In mathematics we begin to provide an intuition of the infiniteness of the number system. The presentation of the names for the giant numbers at some enlarges this concept as the child begins to discover that, though we give names to greater and greater numbers, there is always that number plus one more. We can introduce, too, the various series of infinite integers and, when the child has had set work, Georg Cantor's one-to-one correspondence as a clue to the infinite set.

Mathematical theory leads us directly into the field of geography/astronomy where we find great possibilities for the contemplation of the infinite. We know, for instance, that the 200-inch telescope can penetrate over a billion light-years in every direction, and as far as we can see or photograph, galaxies stretch out. If we imagine ourselves at the center of a sphere, the radius of a billion light years, we have the "observable universe." Our galaxy contains about a hundred billion (10^{11}) stars. In the observable universe, it is estimated that there are about a hundred billion galaxies. Our imagination must again take over as we approach the expanse of this universe; and the limitations that the term "observable universe" implies. In this connection, an investigation into the experiences of the astronauts is a good one for striking the imagination. And the exploration of this infinite space of which we are such a small part is one which the children can relate to the real progression of events that makes our age a continuing history in the making.

We can give impressions of infinity through music and art also. Of particular note is Charles Ives "The Unanswered Question," a contemporary piece with strings representing the infinity as a constant sound pattern while the brass and flute exchange the question and answer that is finally unanswered. In Vivaldi and Bach we have the infinite mathematical progression in the musical composition. In contemporary sculpture we find interesting examples such as Alberto Giacometti's "Hands Holding the Void" Here the woman holds the void, but the stars of her large wide eyes contemplate the nothingness before her. We may consider the work of the American Louise Nevelson. The work of Miro who portrays the idea of infinite space beyond the figures. The work of Pollock who gives the idea through his painting that the painting itself can continue into infinity with the whiteness. The work of Eastern artists is a particularly interesting study in the depiction of space. The intervals of space are as important a consideration here as the figures. The rhythm of spaces is important. Fog is often the metaphor for space penetrating everything. But the sense of the figure within the infiniteness of space is a particularly powerful vision in the Oriental's art. As a contrast, we can see in Western art a need to fill everything. And no real consideration of the design of the space between things.

Perhaps this is a result in part of the history of Western thought regarding infinity which is another interesting study for the child as he meets the great civilizations and explores the culture's attitudes as expressed through their mathematics , their astronomy, their geometry. Euclid always used the line segment extending as far as necessary in either direction; but he avoided the infinite. In Greek science the concept of infinity is scarcely understood and frankly avoided. Thus they preferred circular motion, the study of the sphere, etc. Aristotle: "the infinite is imperfect, unfinished and therefore unthinkable. It is formless and confused." And so only as objects were distinct and defined did they have a nature and meaning. Sophocles: "nothing that is vast enters into the life of mortals without a curse." At the same time, the Greeks saw the study of mathematics as a mental preparation to pass from the world of matter to the world of the spirit. That it was the duty of the intelligent man to use his mind for the contemplation of the eternal ideas because the senses grasped only the passing and the concrete. Plato seized upon this idea and says: "Light from the highest realities, residing in the divine sphere, blind the person who is not trained to face it. He is like one who lives continually in the deep shadows of a cave and is suddenly brought out in the sunlight. Mathematics is the ideal means to make the transition from darkness to light. It is an interesting contrast. And only one example of the many historical references we can note that expand the concept of infinity throughout the ages.

Galileo said that "infinity and indivisibility are in their very nature incomprehensible." And so surely we cannot hope to offer the children conclusive experiences on the topic. Yet, as their experiences with the idea of infinite space and infinite time intertwine, their intuition of infinity expands, their mental understanding grows, their imagination becomes keener. We note finally the symbol invented by John Wallis of Oxford in 1700: . . .and so on. . .to infinity.

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