THE DECIMAL SYSTEM/THE FOUR OPERATIONS

The child is now ready to approach the four operations of the decimal system: addition, subtraction, multiplication and division. Each of these operations can be carried out in two ways: static - without carrying, and dynamic - involving the process of carrying over. In the elementary school, we present and work with dynamic addition and dynamic multiplication. The presentation of subtraction is static, but the children immediately begin to work with dynamic subtraction. For division, we work both with static and dynamic. Before proceeding to introduce the operations, we must teach the child what it means to "carry over."

HOW TO CARRY OVER: A PREPARATION

Direct Aim: To prepare the child for the operation of carrying over.
To learn the principle that no quantity of any category can exceed 9, thus creating the necessity of the carry-over process.

Materials:

1. The bead bank---a shelf supply of decimal system beads: golden units, tens, hundreds, and thousands.
2. The number bank---a table display of the large numeral cards in order.
3. On the table for presentation: a pile of many unit beads, one of ten-bars, one of hundred-squares, and a few thousands.

Presentation:

1. The child counts the unit beads, stopping at ten.	1. We are going to learn how to carry over. First will you **count** the unit beads here?
2. Each time the child counts ten, he exchanges them for a 10-bar at the bead bank.	2. We cannot have 10 loose beads. When we come to 10, we have to change the units for a ten. Will you bring one ten from the shelf in exchange for the ten units?
3. The child places the 10-bar near the units and continues to count, bringing a ten-bar for every ten he counts. He places the remaining units next to the tens he has brought.	3. What have you done? Why haven't you changed these? How many units did you originally have here? What happened to the others?

4. Move the 10-bars over to the pile of tens.	4. We can't leave these tens with the units. We have to put them together with the tens. They have become (2) tens.
5. The child now counts the 10s and the 100s, following the same pattern of exchange and carry-over.	
6. The child brings the numbered cards for each of the four quantities.	6. Now will you bring the right number cards for the amount we have in each pile? How many units will you bring? How many 10s, 100s, 1000s?
7. Child places the cards on the corresponding piles.	7. The quantity we have now is the same, but it has been organized in another way, according to the rules of the decimal system. The rule says: there cannot be more than 9 of anything.

ADDITION

Direct Aim: To give the function of the operation addition.

Material

1. The bead bank.
2. The large numeral display: used for the sum.
3. Three series of small numeral cards displayed in order on a separate table: 1-9, 10-90, 100-900, 1000, 2000, 3000 in every set. Small numbers used for the numbers added.
4. Wooden trays with small trays for units.
5. Large square cloth.

Presentation

1. Give three children a tray and ask them to take a combination of symbols (small numerals) on their trays.	1. Will you make a number on your tray from these small cards?
2. Children put on their trays the corresponding bead quantities.	2. Now take the beads you need to make the number you have chosen.
3. The teacher acts as cashier, counting the quantity on each tray to make sure the number of beads corresponds to the chosen symbol.	
4. Each child takes his symbols off the tray and makes it into a whole number.	4. Will you take your number cards now and make your whole number together?
5. Teacher places the three numbers one under the other.	5. We place our three numbers carefully one under the other like this.
6. Create the visual impression of addition: Teacher puts all the quantities together on the square cloth and makes a pouch containing all the beads together.	6. Now we are going to do a wonderful thing. Watch. We'll put all our quantities together. We are going to do something called addition. The word addition comes from the Latin word which means to put together. Look at the three numbers you brought. We are putting these three numbers together.

7.	Teacher reveals the beads all together. The children separate the categories.	7. Now that we have put all the quantities together, let's see what we have. First we must put all the units together, all the tens, the hundreds and the thousands.
8.	Children count the quantities. When necessary they do the exchanging for the carry over.	8. Now let's count to see how many we have together. Will you count the units for us first? That is ten units. What must you do now?
9.	When less than 10 are left in the category, a child brings the large numeral card for the correct number of units, tens, hundreds, thousands.	9. Bring now the big numeral card for the number of units we have. What shall we put here if we have none?
10.	When all categories have been counted and identified with the symbols, a child makes the number together.	10. Will you make these now into our number? Will you read the number you have made?
11.	Teacher places the +, =, and the ___ around the numbers to indicate the operation. Finally places the sum.	11. These are the signs of addition. This is the sum of our addition, the number that tells how many we have when we put these three together.
12.	Child copies the addition in his notebook.	12. When you write this in your notebook, watch carefully to write the units in a line, exactly one under the other; and the tens, the hundreds, and the thousands.

SUBTRACTION: THE IMPRESSION: A SHORT COMEDY

Direct Aim: To dramatize the function of subtraction: one person has a large quantity and then a person who has nothing takes something away.

Material

1. One tray prepared with a large quantity in beads and the corresponding large numerals.
2. Three trays prepared with no beads, but each with a number made of the small numerals, the total of which equals the quantity on the first tray.

   Example: 7968 (First tray, large numerals)
   3214
   1533 (Three trays, small numerals)
   3221

   This must be a static subtraction.

Presentation

1. Teacher holds the tray with large quantity, and gives the three trays to three children.	1. I am a very rich man with alot of money. And here are three poor children.
2. Teacher counts each category on the tray to verify the numeral and then puts the numerals together to make the number and sets it to the side of the tray.	
3. First child comes and takes the quantity in beads indicated by the number on his tray.	3. You, poor child, may come and take away the quantity of your number.
4. Teacher counts what is now left on his tray. And then gets the corresponding large numerals.	4. Now I have much less money.
5. Second child and then the third each take their indicated quantities.	5. My money has become so little. Now I have nothing. You children have taken it all away. I am a poor man.

PREPARATION FOR SUBTRACTION

Direct Aim: To give the idea that in order to take away, sometimes we must break the quantity apart.

Material

1. One tray with a 1000-cube on it and the large 1000 numeral card.
2. Another tray with only the small numeral cards 999.
3. The bead bank.
4. The large numeral display and the small numeral display, at least one series set out.

Presentation

1. Teacher holds the tray with 1000, and gives the child the tray with the small number 999. Places the large numeral card 1000 on the table.	1. I have 1000. Come and ask me for the number that is on your tray. It will not be easy to give you 999. How can I do it? I must change this quantity. If I have 1000, how is it formed? How many hundreds?
2. Child exchanges the 1000 for ten 100s.	2. Then go and change this 1000 for ten 100s.
3. Teacher counts the 100s.	3. We should count them: 100, 200, 300, . .1000! We have 1000 again. How I can give you 900s. But you want something more. What do you want now?
4. Child changes one 100 for ten 10s.	4. How can I give it to you?
5. Repeat the process for tens and finally distribute the 9 units.	5. I will count these units out, and I can give you 9. I am left with only 1.
6. Teacher places the small 999 below the 1000, which she has placed already to the side. Then places the -, the = and ____. Finally the small numeral 1 showing the difference.	6. These are the signs of a subtraction. In addition we put things together. In subtraction we take one quantity away from another and we get a smaller number. What did I have left? That is our answer---1.

MAKING A SUBTRACTION

Direct Aim:
1. To give an understanding of the concept of subtraction: to take away.
2. To make sure the child understands that a quantity of the next hierarchy can be broken down into parts to be added to the lower hierarchy.
3. To give the function of the operation subtraction and its check.

Materials
1. The bead bank.
2. Ordered display of large numeral cards.
3. One series of small numeral cards in an ordered table display.
4. Wooden trays.
5. Box containing operation signs.

Presentation

1. Teacher writes a subtraction problem on the blackboard or on paper. Dynamic subtraction. Example: 4392 - 3878	1. We are going to make this subtraction.
2. One child takes on his tray the bead quantity and the large numerals for the minuend.	2. Will you be the rich man? On your tray take the bead quantity for this large number and the big numeral cards.
3. Second child takes only the small numeral cards for the subtrahend.	3. You will be the poor child. Put on your tray only the small numerals for this number that you are going to take away from the rich man.
4. First child counts his quantities, makes his numerals together and places the number on the table.	4. Count your money now, rich man. Show us how much you have all together. Will you place that number on the table?
5. Second child places his small numerals under the large ones, and one then places the correct subtraction signs to show the operation that is taking place.	5. Now will you put your smaller number under that one to show how much you are taking away? We must place the subtraction signs here, too, to show what operation we are doing. Where do they go?

6. Second child asks for his number, beginning with units. If the rich man does not have sufficient units, the second child exchanges a 10-bar for ten more.

6. Our poor child may ask for his number and take it away. He must ask first for the units.
Are there enough?
How can he make more units so that the rich man can give him the right number?

7. When the exchange is made, the first child counts them all and gives the second the number he requests.

7. Count your units.
Now you have enough to give him what he needs.

8. The process repeats: the second child requests 10s, 100s, 1000s, changing the necessary quantities. The first child counts them out and gives the correct amount.

9. **First child counts what he has left**, takes the small numerals for each quantity, and makes the difference number, placing it below the line.

9. How much do you have left?
Count it and then get the small numerals to show that number.
Now make the magic number and place it here below the line.
This is the number we have left, our answer.
What is it?
Why did we make it with the small numerals?*

10. Teacher shows how to check the answer. Puts the two small bead quantities on one tray together. On the table arranges the two small numbers together with the addition sign. Puts the large number aside, separated into the 4 numerals:

4000

300

90

2

10. Now we can make a check: We'll put the quantity we took away together with the quantity we had left.
We show the quantities have been put together by making an addition with out small numerals.

4392
- 3878
514

3878
+ 514

11. Children count units, tens, hundreds and thousands, making the necessary changes whenever 10 of one category is counted.

11. Let's count how many we have together.
How many units are there?
What must we do with these 10?

11. After each category numbers less than ten and the count is made, child places the large numeral, already on the table, on that category.

12. When all categories have been counted, child makes the large number together from the four numerals, and places it under the line.

3878
\+ 514
= 4392

11. Place the unit card we have here on the units you have counted.
Are they the same?

12. Make our big number together now and place it under the line.
Together the number we took away and the number we had left make the big number that our rich man originally had.*

Control: Putting together the subtrahend and the difference to show that their sum is the minuend.

Age: After 6.

* NOTE: When the result of the subtraction problem has been placed, the children should copy the whole problem in their notebooks. Again, when the check has been finished, they should copy the addition in their notebooks.

MULTIPLICATION

Direct Aim: To give the function of the operation multiplication.

Material

1. The bead bank.
2. The large numerals display.
3. The three series of small numerals displayed in order: each series with the numbers 1-9, 10-90, 100-900, 1000, 2000, 3000.
4. Wooden trays.
5. Several blank square cards on which the multiplier is written during the presentation.

Presentation

1. Give three children empty trays and ask them to bring the same number from the small numerals. (three series)	1. Each of you must choose from the small numeral cards the <u>same</u> number, and bring it on your tray.
2. Children take the corresponding bead quantities on their trays and place the small numerals on top of the quantities.	2. Now take the bead quantities to show your number. Place your small numerals on top of each bead group.
3. Teacher counts quantities on each tray to be sure they are the same. Counts 1000s first, then 100s, 10s, units.	
4. Proceed as in addition.	4. We are going to put all these quantities together now like an addition.
5. One child places the three small numbers (symbols) on the table, showing addition and places the addition signs.*	5. Will you place our number cards here to show the addition we are making? We will need the signs of addition, too.
6. Second child places all the beads together on one tray and counts them. The third child helps by making the necessary changes for the "carry over" operations.	6. Will you (two) put all the beads together and count them, beginning with the units? Remember that when we count to ten of one thing, we must change it for something different.
7. As the children finish counting each category, one brings the large numeral card to show the total.	7. When you have less than 10 units, count them and bring the big numeral card for that number. Then, as you count, bring the large numerals for the 10s, the 100s, and the 1000s.

8. Child makes the whole number with large numeral cards and places the sum below the line.

8. Now will you make the number for the quantities you have counted and place it below the line to show our answer.

9. Explain that multiplication is an addition with the same number.

9. What have you done?
What is the difference between this addition and the ones we usually do?
This is a special addition.

10. Show how it is written and point out the function of the multiplier, writing it on a blank card and placing it in position.

10. It is also written in a special way.
How many times did you bring the same number?
So we can use one number and put away the other two.
We will write 3 on this card to show how many times we take this number.
We put it down this way:

```
  2418
+ 2418
+ 2418
= 7254
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2418 X 3 = 7254

You have done a special addition that has another name. It is called multiplication.
This X is the sign of multiplication.
This 3 tells me how many times I have taken this number.
We say: 2,418 taken 3 times is 7,254.

11. Point out that when the children do this exercise together, each child must bring the quantity in beads, but only one person needs to bring the numeral cards.

11. When you make a multiplication together, each one of you must bring the bead quantity for the number you choose.
But only one person needs to bring the numeral cards.
If two children had brought the same quantity, what number would you write on this card.

12. Emphasize the function of multiplication.

12. Multiplication is an addition with the same number.
You make a ticket that tells you how many times to take that number.

DIVISION

Division is characterized by the fact that the same quantity is separated into EQUAL parts, as many times as we indicate.

When we make a subtraction, we separate a quantity into separate parts, but these two parts do not have to be equal. Like addition, the quantities do not have to be the same. In subtraction we are taking a different quantity from the first. If we take the same one, nothing is left. The quantity we take away can be smaller or equal, but never greater.

In multiplication, we put together equal numbers. In division, we are separating a number into equal parts.

To make division clearer, it is preceeded by an active operation with the materials which shows the child that this quantity we have has to be divided into equal parts.

Again, we are only presenting the function of the operation without any calculation. With the decimal system, we are giving the child distributive division which means to distribute equal quantities. We give equal quantities to each child.

There is another important, different division: group division, which we present with other material.

Division is subdivided into 2 parts:

Division with a divisor of one digit.

Division with a divisor of two or more digits.

DIVISION: ONE-DIGIT DIVISOR

Direct Aim: To give the function of the operation division with a one-digit divisor.
To show that the result of a division is what one unit gets.

Material

1. The bead bank.
2. One set of large numerals displayed.
3. Three series of small numerals.
4. Wooden trays.
5. Blank cards on which the divisor is written during the presentation.

Presentation

1. Teacher asks child to bring a large bead quantity on a tray and to bring also the large numeral cards for that quantity.	1. Bring me the quantity 4,627. on the tray. Bring also the big numeral cards for that number.
2. Teacher checks the quantity, counting 1000s, 100s, 10s, units. Then she puts the numeral cards together to make the number and places it on the table.	I have 4,627. I want to give each of you three children exactly the same amount.

3. To each of three children, teacher gives an empty tray and distributes the quantity on her tray equally among the three, starting with the 1000s. Exchanges, when necessary, for ten of the lower hierarchy.

3. I will begin with the biggest. Here is 1000 for you, 1000 for you, 1000 for you. If I give this last 1000 to one, I won't be able to give the other two anything. What should I do?

4. When the units have been distributed, note the remainder.

4. I have one unit left. I can't distribute it equally among three persons. What can we do with this? We can't do anything. It is left over. Since I can't distribute it equally to you, I will keep it. It is my remainder.

5. Each child counts the quantity he received. ONE child goes to bring the small numerals to show the quotient.

5. Each of you count the quantity you have received. Will you (specifying one child) bring the small numerals to show the quantity you have on your tray?

6. Teacher checks the quantity with the numerals on the child's tray. Asks for a verification of the same quantity on each of the other two trays. Then explains the operation division.

6. This child has received 1,542. How many did you receive? Each person has received the same quantity. The thing we have done is called division. We had a quantity and we divided it into equal parts. That is why it is called division.

7. Explain the function of the divisor. Show how the division is written, writing the divisor on a blank card and placing the sign of the division. Finally, explain that the quotient is the amount that one unit gets. Note the remainder, and use another blank card on which to write it.

7. How many children are you? How many parts did we divide our quantity into? We put the numbers together like this:

This number (3) is called a divisor. It tells how many parts we divided our big number into. What is our answer? It shows how much one person gets. Why did only one of you bring the numeral cards for the quantity? We have placed this sign (÷) here to show that we have made a division. And this number (1) is the remainder. It is the quantity I could not distribute equally.

8. Point up the different use of the large and small numeral cards in the four operations, and the reasons.	8. Why did we start with the large numeral in division? What card did we use for our answer? What other operation have we done that started with a large numeral card? What operation starts with a small numeral card? Why is the answer made with a large one? What other operation is like that? Why?

Control of Error: Teacher may check the answer, or
Children can make the proof:

To check division, the children take what quantities they have received and put them together. Then they count them, and with the large numeral cards, indicate how much the quantity is together. This is in fact a multiplication operation, but with the decimal system materials, it is a simple addition.

DECURION DIVISION

Direct Aim: To give the function of the operation division with a two-digit divisor.

Material

1. Bead bank.
2. Set of large numerals.
3. One series of small numerals.
4. Box of ribbons: 4 green (units), 1 blue (ten).
   Also in the ribbon box can be found 1 red ribbon to represent the hundred in centurion division (3-digit), and another green for the thousand in 4-digit division. The function of these two may be noted if the question arises, but are not used in this presentation.

Presentation

1. Call 14 children.	1. Today we need fourteen children to make a division.
2. One child brings a large quantity with the corresponding large numerals on his tray. The teacher counts the quantity and makes the number on the mat.	2. Will you bring the quantity 8,765 and the large numerals for that quantity?
3. Note the lengthy process of dividing 14 ways. Explain the possibility of one child representing 10, and 4 representing the 4 units. Ask the other 9 to return to their seats.	3. I must divide this quantity among 14 children. What am I going to do? I have to give to each child the same quantity. But if I have to give out these 1000s one by one to each of you 14, and then the hundreds, one by one, etc. . .that will be a very long job. Since you are 14, we can play a game. We can say that one of you represents 10 children. Will you be 10 children? How many units do we have in 14? Then we need 4 children to represent each of the units. Then: here is 10, you are 11, you 12, 13, 14. Here is our 14. The rest of you may watch what happens from your seats.

4.	Tie blue ribbon on the wrist of the child who represents 10, green ribbons on each unit child.	4.	What is the color of ten? Then let's show that this child is 10 with this blue ribbon. And what color ribbon should we use for the unit? Now we can see that this child is 10 and these four each one unit.
5.	Begin distribution with the thousands. For every thousand distributed to the ten child, give the unit children one hundred. When there are not sufficient hundreds, a thousand must be exchanged for ten more.	5.	If I give this ten child one thousand, how much must I give to each of the children who represent one unit? I have to give them 100 because 100 is ten times less than 1000. 1,000 for you, 100 for you, 100 for you, 100 for you, 100 for you. I have run out of hundreds. What can I do to get more? I can change 1000. How many 100s are in 1000? 1000 is ten times more than 100. So. . . When I give this ten child 1000, I am giving him 100 for each of 10 children.
6.	Proceed through distribution until there are no remaining 1000s. Distribute 100s. For every 100 given to the ten child, give 10 to the units. Change hundred for tens if necessary.	6.	I have no more 1000s. Now I will distribute the 100s. When I give you (ten child) 100, what will I give each unit? I must give each of the units ten times less--- and that is 10.
7.	When no hundreds remain, distribute 10s.	7.	Now there are no more 100s. I must divide the 10s. When I give this child who represents ten a 10, how much must I give to each unit? A 10 to the ten, a unit to the unit.
8.	If there is a remainder, make note of what is left that cannot be equally divided among 14.	8.	We have one unit left. I can't divide this equally among 14. So I will keep it. It is the remainder.

9. Point out that the answer is what one person gets. To emphasize this have the ten child call the other 9 back and divide his quantity among them, including one part for himself to show the 10 parts.

9. Now we have nothing left to divide.
What is the result of our division?
How many children are you?
But her (ten child) quantity does not all belong to her.
It belongs to 10 children.
Remember that our answer is what one person gets.
We gave ten times more to the child who represents 10; now she will divide hers equally into 10 parts.
Will you call the other 9 children back, and divide your quantity equally among you ten?
Then we will see how much one person (unit) gets.

Our answer is 626.
Is that the quantity one unit child received?

10. One child brings the small numeral cards for the quotient.

10. Will you bring the small numeral cards to show our answer?
We need only one set of numerals because each person received the same quantity.

11. Show the division operation on the mat with the cards, and the divisor written on a blank card. Place division signs and write another card for the remainder.

11. Now we can show the division we have made.
We have divided this large number into equal parts.
Then we'll write 14 on this card to show into how many parts we divided our large number.
This is called the divisor.
Will you place our division signs?
And here is our answer, the quantity that one unit got.
Why did we use the small cards?

8765 ÷ 14 = 626 R.1

12. When the children understand well, we can explain centurion division using one child and a red ribbon to represent 100. This hundred child would receive 10 times that given to the 10; and the ten ten times more than the unit.

Age: After the child understands well one-digit division.

THE STAMP GAME: PARALLEL EXERCISES WITH THE DECIMAL SYSTEM & MEMORIZATION

The stamp game takes its name from the original exercise which was done with actual stamps that the child pasted in his notebook. Because this was expensive and inconvenient and generally resulted in work less than neat, the materials were standardized into wooden stamps.

The exercises of the stamp game provide an excellent opportunity for the child to work by himself with the various operations whereas the decimal system work is done usually in groups.

Material

1. Wooden stamps in the hierarchical colors: green stamps for units on which is printed in black 1, red stamps for the hundreds, blue stamps for the tens, and green again for the thousands.

   NOTE: With the decimal system numerals, all the cards were white and the numerals in the colors. Here the squares of wood are in the colors and the numerals are in black. In addition, all the stamps are printed with the 1, 10, 100, 1000 instead of the various numerals.
   Also in the decimal system the cards are different in size, becoming twice as large for two numerals, three times for the 100s, four times the unit for the 1000s. Here all the stamps are the same size.

2. Four containers, one for each hierarchy.

Presentation

NOTE: The child can use the decimal system numeral cards to form the operations, but the work is time consuming and it is preferable to simple do the work in the notebook.

1. Introduce the stamp game materials.	1. In this game we have wooden stamps to work with. This is the unit. What color is it? This is the ten. . .
2. Show the decimal system cards, a sampling and note the differences between them and the stamps.	2. What are the differences between these cards and the stamps? What color are the cards? What numerals do we have written on the stamps? All of our stamps are the same size. What is the difference in the sizes of our numeral cards from the decimal system?
3. Form a quantity with the stamps 	3. We read this as a whole number: 1,324.
4. Have the child form a quantity.	4. What quantity have you formed?

THE STAMP GAME. . .
Presentation. . .

5. When the material is understood, proceed with an addition.

5. Now we are going to do an interesting addition.

6. Write the quantities on a slip:

$$\begin{array}{r} 4363 \\ +\ 2179 \\ \hline \end{array}$$

and have the child read it and copy it in his notebook.

6. What quantities are we going to add?

7. The child then forms the quantities one at a time with the stamps, placing them in vertical rows under the containers which contain the stamps.

Thousands	Hundreds	Tens	Units
1000	100	10	1
1000	100	10	1
1000	100	10	1
1000		10	
		10	
		10	
1000	100	10	1
1000		10	1
		10	1
		10	1
		10	1
		10	1
		10	1
			1
			1

8. The child puts the stamps together to make the addition.

8. What do we do when we make an addition?
We put the quantities together.

9. Now the child counts the total, beginning first with the units. When he has 10 units, he must replace them in the container and take a 10 which he adds to his row of 10s.

9. We can discover our total by counting the stamps we have together.
What must you do when you count ten units?
Then we may exchange them for one 10 and add it to our row of 10s.

10. Child counts, tens, hundreds and finally thousands.

11. When he has reduced all rows according to the law of the decimal system, he can read his total quantity: 6542.

12. He writes the answer in his notebook.

THE STAMP GAME. . .

SUBTRACTION

Presentation

1. The teacher writes on a slip a subtraction: 3317 / - 2995 and says nothing more.

2. Asks the child to form the subtraction operations:
To find out if the child knows that he has to form only the minuend---and then from that quantity, take away the subtrahend.

3. The child carries out the subtraction with the stamps:
He takes the subtrahend units first---5---bringing those five to a position on the mat below the first quantity.

4. Next he must take 9 tens from 1 ten---so he must make an exchange.
He puts one 100 back into the container and adds ten 10s to his row of 10s.
Now he can take away 9 from the 11 tens on the mat. He brings those 9 down.

5. Continue taking away the 100s and 1000s, performing the necessary exchanges.

6. When the subtraction is completed, the child counts the remaining stamps and writes the difference in his notebook.

7. Then he makes an addition, putting together the subtrahend and the difference and counting the total.

NOTE: As the exercise progresses, the subtraction should contain first one zero in the minuend, then two, then three---this enables the child to quickly understand the process of the exchange.

MULTIPLICATION

This exercise with the stamp game is not often used.

The presentation proceeds as for addition. The difference is the way in which the operation is written. The child writes: 2654 X 4 =

Then he lays out the quantity 2654 four times and counts the stamps to get the product, which he writes in his notebook.

THE STAMP GAME: Division

Material

1. The same: four colored bowls of wooden stamps.
2. A box of skittles: containing 30 skittles of 6 types:
   a) 9 small green skittles for representation of units.
   b) 9 small blue skittles for the tens.
   c) 9 small red skittles for the hundreds.
   d) 1 large green skittle to represent a thousand.
   e) 1 large blue skittle for ten thousand.
   f) 1 large red skittle for a hundred thousand.

   The skittles represent the divisor. . .that is, the skittles substitute for the children in the decimal division.
3. Green, red, and blue discs---also contained in the skittle box---to be used when there is a zero in the divisor. There might be 1, 2, or 3 zeros.

Division with 1-Digit Divisor

The only difference in the division exercise here is that instead of the group which was needed to represent the divisor in the decimal system work, the child can work by himself. He can use bigger divisors with no trouble. . .and he can use zeros.

Presentation

1. Write on a slip the division:	1. 2,114 ÷ 7 =
2. The child lays out the stamps to show the dividend.	2. What is the quantity that we want to divide into equal parts? Will you show that with the stamps?
3. Introduce the skittles, setting them out to the right side of the bowls---the position where the quotient will always be shown in columns. DO NOT SHOW THE LARGE SKITTLES AT THIS POINT.	3. Instead of children in this game, we now have these skittles to show how many parts we divide this quantity into. This small green skittle represents one child. How many will the blue skittle represent? This red one represents 100 children.
4. Place the number of skittles needed to show the divisor in a row horizontally to the right of the bowls of stamps.	4. Into how many parts must we divide 2,114? Then we should place seven unit skittles here, to show the divisor.
5. Proceed with the division, exchanging the stamps for ten of the lower hierarchy when necessary.	5. What do we begin with in our division? We start by dividing this quantity into equal parts, beginning with THE LARGEST QUANTITY.

STAMP GAME: Division. . .
Presentation. . .

5.	. . . the division	5.	I cannot distribute these 2 thousands because I have seven children. So I must change each one of them for 10 hundreds. Now I can begin distributing the hundreds. . .
	The child lays out the exchange in vertical columns under each dish. . .in this case, below the hundreds.		
6.	Continue through the distribution of all the quantities. Emphasize that the answer is what one skittle gets.	6.	Now I have distributed everything. What is our answer? 302. The result is what one person---here, one skittle---gets.
7.	The child writes the quotient in his notebook, noting the remainder if there is one.		

Division: Division with a Two-Digit Divisor

1.	On a slip, write: the division.	1.	728 ÷ 23 =
2.	The child lays out all the material for the division. . . note the correct skittles to be used for the divisor.	2.	Which skittles will we use to show our three units? How many tens must we show? Which skittles shall we use for the two tens?
3.	Begin the distribution, noting that the tens must always receive 10 times more than the units.	3.	I must begin by distributing the hundreds. If I give hundreds to the two tens, I must give the units ten times less. And since I have three units, an d only 2 tens, I must begin by exchanging one of the hundreds for ten 10s.
	Exchange when necessary. . . here at the beginning.		
4.	Continue with the distribution, always making sure that the tens receive each time ten times the units. . .note the remainder.	4.	Now I can give the tens one hundred and each of the units one ten. When only 1 ten and 5 units are left, we cannot distribute anything further. 15 is the remainder.
5.	Stress the quotient as what one UNIT receives.	5.	How do I know what the answer is? It is what ONE UNIT gets: 31 R. 15.
6.	Note that the remainder is smaller than the divisor. . .and complete the rule.	6.	Is the remainder bigger or smaller than the divisor? When the remainder is smaller, it tells me that I cannot go on distributing. As soon as I have a remainder less than the divisor, I cannot continue. The remainder cannot be euqal to nor greater than the divisor.

AGE: 7 years

STAMP GAME: Division with ZERO in the Dividend: 3 - Digit Divisor

4072 ÷ 134 =

Presentation

1.	Introduce centurion division.	1.	Remember when we made a division with two digits? We gave the units ten times less than the ten. That was DECURION division. Now we have CENTURION. This red skittle (small) shows 100 children. We have here in the divisor, 134 children who must receive an equal quantity.
2.	Point out that the skittle representing 100 must receive 10 times more than the ten skittle. Begin the distribution with the thousands---make the necessary exchanges. The child exchanges one thousand for ten hundreds.	2.	If I give the skittle representing the 100-child 1000, how many should I give the 10-child? I must give that blue skittle ten times less. If I give the red skittle 1000, I must give each blue skittle 100. But how many hundreds do I have? What must I do? Now I can give each of the three tens 100. And how many shall I give each unit? Each unit gets ten times less than the ten. So each of the four units will receive ten.
3.	Continue the distribution until the remainder is less than the divisor---or until a complete distribution is impossible.	3.	NOW---if I give these 10s to the hundred, I can give units to the tens, but I will have nothing to give to the units. So I must stop. The rest is my remainder.
4.	An IMPORTANT CONCEPT here again: the result is what one unit got. Each unit received 30. Each child represented by the blue skittle still got 30: 300 ÷ 10 = 30 and the red skittle receives 3000 ÷ 100 = 30	4.	What is our answer? What one unit received. What did one unit receive? Why did each blue skittle receive 300? Then each child represented by the blue skittle received 30. How many children are represented by the red skittle? Then we must divide this 3000 among those 100 children. What does one child receive? We have divided the quantity 4072 into 134 equal parts --- and each part is equal to 30.

Note that the remainder, 52, is less than the divisor.

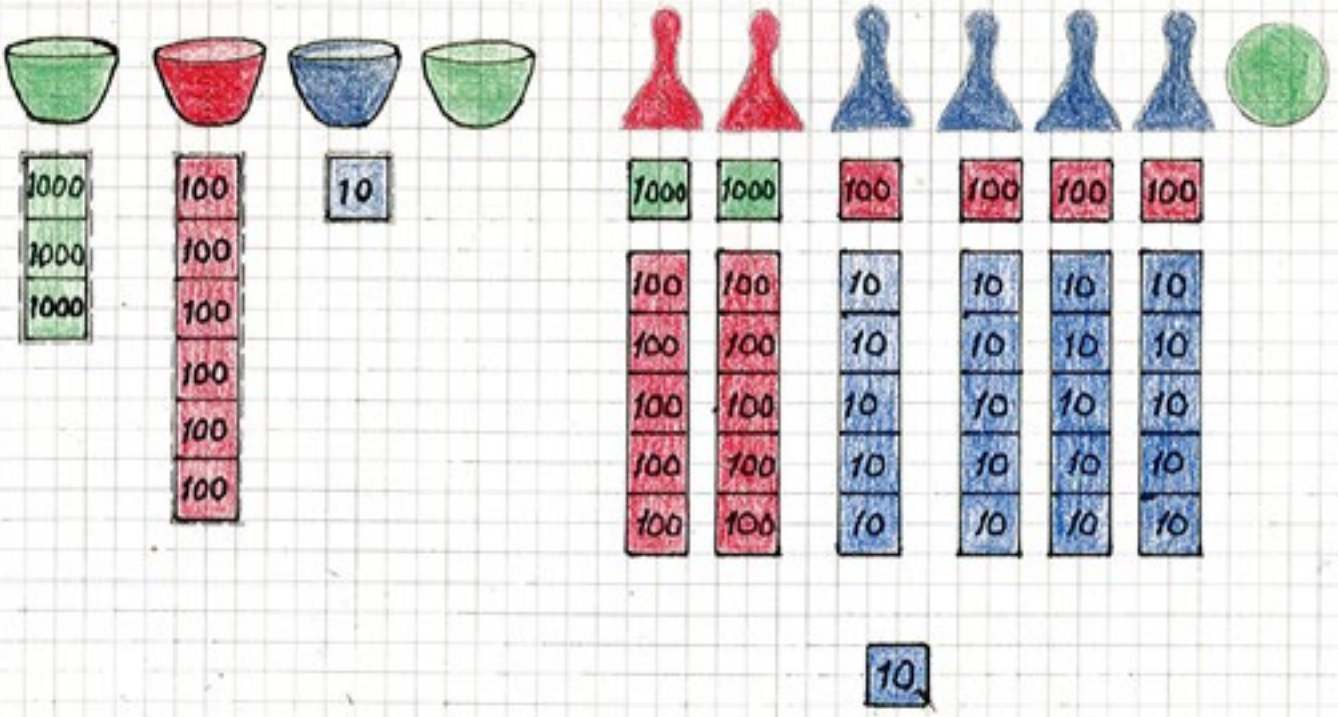
What is our remainder? Why is it so large?

STAMP GAME: Division With Zero in the Divisor and the

Presentation

1. Write on a slip: The child shows the dividend and then the divisor with the skittles, using the green disc to show the zero in the tens position.	1. 4072 ÷ 104. First we form the dividend with the stamps. We need one small red skittle to show the hundred. To show that we have no tens, we place the blue disc here. Then we need four green skittles.
2. The child makes the distribution; point out that the zero only holds a place and therefore gets no part of the dividend.	2. Where do we begin our distribution of the dividend? So we will give the hundred a thousand. But here I have zero tens. I don't need to give the zero an equal part because THE ZERO ONLY HOLDS THE PLACE. It is no quantity. If I have zero children here, then I don't need to distribute anything. Now we must give the units ten times less than the tens would have received. (20 times less than 100s) How many will the units receive? Each will receive a ten.
3. The child continues the distribution. He makes the necessary changes---and encounters the need to exchange two categories to make the distribution---a two-step exchange.	3. Now the hundred gets another thousand. . . and the units must again receive a ten. But I have only three tens. . . and I need four. How will I get more tens? I have no hundreds. Then I must exchange one thousand for ten hundreds. . . and one of those hundreds for ten tens. Now I may continue with the distribution.
4. Finish the distribution. Note that the answer is what one unit gets.	4. What did each unit get? 39 And what is our remainder? 16

3,610 ÷ 240



= 15 r. 10 (150÷10)

Division: Zero in Another Place

Presentation

1. 4072 ÷ 120 Child shows the problem with the materials.	1. This time we must put the green counter in the units position. (as shown above)
2. Proceed with the distribution. When there remains only 1 hundred, 1 ten and 3 units, the distribution cannot proceed further because:	We cannot give tens to the hundreds-----and units to the tens because those quantities would be impossible to distribute. 10 would be impossible to distribute among 100. . .1 impossible among 10.
3. We have a very large remainder that should be noted.	3. Why do we have such a large remainder? Is it smaller than the divisor? How shall we show it?
4. Note that the answer is what one unit gets. . .if the child does not see why one unit gets the ten's quantity divided by ten, have him divide it on paper: 330 ÷ 10	4. Our answer is what one unit gets. But we have no unit. How will we discover the part that one unit gets? To find out what one unit gets, we must divide the quantity that the ten received by 10.

STAMP GAME: Distributive Division and Group Division

The difference between distributive division and group division is in the verbalization of the problem. Thus the problem

$$672 \div 6 = 112$$

can be read: The quantity 672 can be distributed into 6 equal parts and each part is 112. That is the dividend is distrubted equally among 6 persons and each person receives 112. This is DISTRIBUTIVE DIVISION.

or it can be read: The quantity 672 can be divided into groups of 6; and the quotient will represent how many groups there will be of that divisor quantity. In this case, there will be 112 groups of 6. That is. . . "How many times is the 6 contained in 673? How many sets of 6?" This is GROUP DIVISION.

The concept of group division is a very important one because when we carry out division abstractly, we always operate with group division.

Presentation: Group Division

1. Write a simple division. The child forms the dividend with the stamps.	1. $24 \div 3 =$ First we will show our dividend with the stamps.
2. The child then forms as many groups of 3 as he can with the dividend, necessitating an exchange to units.	2. Instead of showing out divisor with skittles, this time let's see how many groups of 3 I can make with 24. We must first change the 10s into units because with the ten we cannot make groups of 3.
3. The child puts the units into little stacks of 3---and discovers he can make 8 when he counts the stacks.	3. Now we have 24 units. How many groups of 3 can we form. We have made 8 groups. This means that 3 can be found in 24 8 times.
4. The child writes the answer. Note that this has been group division.	4. We have discovered that there are 8 groups of 3 in the quantity 24. Our answer is 8. 24 divided into groups of 3 is 8 groups. And we have demonstrated GROUP DIVISION.
5. With the small stacks still on the mat, note that division is a subtraction of equal quantities.	5. We know that division is a subtraction of equal quantities, just as multiplication is an addition of equal quantities.

STAMP GAME: Group Division. . .
Presentation. . .

6. Have the child write the series of subtractions for $24 \div 3 = 8$. . .and as he does so, he can remove the groups of 3 one at a time---8 times.

6. When I say $24 \div 3$, it is the same as $24 - 3 - 3 - 3 - 3 - 3 - 3 - 3 - 3$. Let's write those subtractions. As we subtract 3 from 24, we will remove one of our small stacks.

$$\begin{array}{rl} 24 & \\ -\ 3 & \\ \hline 21 & \\ -\ 3 & \text{Remove a stack} \\ \hline 18 & \\ -\ 3 & \text{" " "} \\ \hline 15 & \\ -\ 3 & \text{" " "} \\ \hline 12 & \\ -\ 3 & \text{" " "} \\ \hline 9 & \\ -\ 3 & \text{" " "} \\ \hline 6 & \\ -\ 3 & \text{" " "} \\ \hline 3 & \\ -\ 3 & \text{" " "} \\ \hline 0 & \end{array}$$

How many times did we subtract 3 from 24 to reach zero?
Then 24 can be divided into groups of 3 eight times.
We subtracted 8 groups of 3.

NOTE: We are proving here that there are really only two operations: addition and subtraction.

7. If the child is particularly interested, we may also have him observe, with the stamp materials shown as a distributive division, that we may read the group into which the dividend is being divided across the row. That is, when we have divided the equal parts once to the divisor, we have shown one group----AND we have subtracted that group from the dividend. THIS IS AN ADVANCED CONCEPT. . .and should be reserved only for the child who really wants to know. In this concept of group division, the answer is again how many groups are distributed.