

## MATHEMATICS: AN INTRODUCTION

An understanding of mathematics is an essential component of the child's ability to adapt to his environment. We know that adaptation is the work of all living things; that the man who is not able to adapt will probably go on living, but without joy. And so it is our concern as educators and helpers of life that we aid the process of adaptation that the child is making. In a world constantly more highly technological, more mathematically oriented, more scientifically complex; mathematics enables man to adapt to modern living.

We must approach mathematics for the child recognizing that it is an abstract for him, a system which he must learn; in contrast to language which is a natural function of the child. We must remember, too, that mathematics is increasingly complex, that it represents the labor and investigation of men for centuries---and that the child is expected to learn all of this in a short time. While mathematics becomes increasingly important, it also becomes more complex and harder to understand.

Our approach to the study of mathematics, just as our work in other areas of the school environment, must be as interesting and natural as possible. School is not a natural environment for the child; too often it obliges him to do work against his nature. Our task is to transform that environment into one that bears a real and captivating relationship to real life.

And, in fact, mathematics is a very real part of man's life. It has developed with life itself. The origins of mathematics are not abstract, but rather concrete and practical. The children must understand that mathematics developed as a result of an expression of man's need, and that without it, one cannot live in society. Counting began with the sense of property, that is, "what is mine?" and "what is yours?" Always for men it has been important to know how much he possessed. At first he had no numerals. There were common things in the history of every people that represented how much was produced and what exactly people possessed. We know by these systems a great deal about the history of a certain group. And we find their numerical codes a real expression of the intelligence of man at that point. The more complex the system, the more advanced the civilization.

Through these various groups, there was born diverse systems of numerations. The origin of geometry was a practical one. It was born with the annual overflow of the Nile in Egypt. Every year, when the flood waters receded, the land around it reappeared and it was necessary to remeasure the land and reestablish the boundaries. The first tool of measurement was a chain of 100 rings. Later a short chain of perhaps only three or four rings still represented 100 after the original length. On the river there were many water lilies; eventually a drawing of one water lily came to represent 1000. And observing the myriads of tadpoles, a drawing of one tadpole began to represent 10,000. Thus we see developing a system of symbols.

And it is an especially important concept that numbers are, in fact, symbols. We are surrounded by symbols which society has decided upon and given a certain value. Our system of numbers has no real value, but simply the value that men have agreed to give to them.

The development of these symbols during the course of history is a fascinating progression. Ancient peoples accomplished great mathematical feats with the simplest of instruments. The Egyptians built the great pyramids, knowing only the triangle with sides respectively 3, 4, and 5. It is something of a wonder that they were able to work out the formula for the area of the truncated pyramid. In geometry they did have operating formulas for the area and volume of elementary figures and they calculated the value of "pi" as 3.16. In arithmetic they could add and subtract whole numbers and used the same additive method to multiply and divide. They used a decimal system.

The Sumerians counted to 2, and their numerical system was thus binary, based on 1 and 2. It is interesting that modern computers go back to this binary system. Another ancient system was a system of 12, based on the twelve parts of the four digits on one hand. The decimal system is based, of course, on the fingers of two hands. However, with any system there was much difficulty with complex operations until the Hindus discovered the zero. Suddenly it became possible to count to infinity. In 800 B.C. the Indians take their system of numbers to Baghdad; and the Moslems begin to disperse the information throughout the known world. It was the Arabians who founded the first great universities. The Greek contributions to the development of mathematics was great. Thales and Pythagoras founded the science of geometry. The Pythagoreans distinguished many numbers, worked out arithmetic and geometric progressions, compiled tables of square and cubes. Euclid produced his famous and still used mathematical laws in The Thirteen Books of Elements. We can begin to see the total effort of humanity towards the achievements of our present mathematics system. In three centuries of work, the intelligence of man compounds and becomes always greater. And so, knowing how much more the child must learn today, we must marvel that his mind is able to comprehend in such a short time all the complexities that represent the conquests of history.

In order to achieve this understanding, the child must first have concrete experiences that involve much practical work and experience. Through this experience, the child comes upon understanding. And finally, he is able to make the abstraction. Just as humanity first encountered mathematics, so we must offer the child first practical application, and then the rules. If we present abstraction first, the child throws up a barrier and does not even really understand even though he may learn the rule. Thus we must approach our study carefully. We must keep the child's interest alive. We are equipped with the Montessori materials which open the mind and allows the child himself to come happily upon understanding. They offer indirect preparations which offer the children the possibility to have their own experiences and thru them, by means of the subconscious which stores that experience, to come to understanding naturally. To accumulate slowly concepts which suddenly surface to the conscious and become the child's very own. The Montessori materials are, indeed, the child's food---and we discover that for mathematics he is very hungry.

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