MEMORIZATION

Memorization is the process of fixing in memory experiences and knowledge. It is important both methodologically and psychologically.

There are two ways to make a child memorize. The first method is mechanical and, for the child, boring. The second method is one of parallel exercises, organized activities, each different, but having the same aim, so that as the child moves from one exercise to the other, he retains interest. He may do several of these parallel exercises on the same day, and each one reinforces the same idea. Memorization thus begins to occur spontaneously. In these parallel exercises Dott.ssa Montessori emphasizes the role of the work of the subconscious. In this storehouse of experience and knowledge, the child accumulates, through the exercises, a certain amount of memorized information which he can use.

The decimal system materials related to memorization. Through that material the child got the function of the operations. Now with the exercises in memorization, one detail is examined. When the child worked with the beads, his progress was hindered because he didn't know by memory the combinations. When he placed 9 together with 9 more, it was necessary to count the beads before he knew that their sum was 18. Now the teacher must keep his interest alive long enough for him to memorize such combinations so that he can go on. And when he later confronts 9 + 9, he knows that it is 18.

Just as the child must learn to read to enter the world of culture, so he must memorize, for therein is the key to the world of numbers.

We begin, then, with an important detail: the memorization of all the possible combinations of the digits 1-9 using two addents. We will show that it is sufficient to know these combinations in order to solve any addition; and, furthermore, that there are only a certain number of these combinations.

COMBINATIONS OF THE DIGITS 1-9 WITH TWO ADDENTS

Direct Aim: To memorize the result of any possible combination of the digits 1-9 using only two addents.

Indirect Aim: To prepare the child for making all possible combinations in addition.
To prepare for understanding and using the commutative law.

Materials

1. The addition board.
2. A box of wooden strips, a blue set representing the quantities 1-9 and a pink set representing the quantities 1-9.
3. Combination booklets: printed forms with pages of all the combinations of 1-9 with two addents.

MEMORIZATION materials. . .

4. Box containing the same combinations as the printed forms, but each one cut out separately in small strips.
5. Charts 1-4.
6. Wooden box containing set of wooden stamps.

Presentation

1. Introduce the addition board, laying out below it the pink and the blue stair.

2. Show how an addition may be made on the board, and how the answer is read.

2. We always place the blue strip first. We look for the answer above. Now we see that 7 + 5 = 12.

3. If the child is interested in the red line, explain that it indicates how many more than 10 we have added.

3. The red line shows us how many more than ten we have added. So we can see that 10 + 2 = 12. We can count the two pink squares very easily.

4. Introduce the combination booklet and Chart #1. Then explain the Exercise #1.

EXERCISE #1

Materials

1. The addition board.
2. The two sets of wooden strips.
3. The combination booklet of addition problems.
4. CONTROL - Chart #1 with all the results. (10 written in red)

a. The child reads the first combination in the booklet, page one, which begins 1 + 1 = . He places the blue 1 and the pink 1 on the board, and reads the answer: 2.

b. He then writes the answer 2 in his booklet.

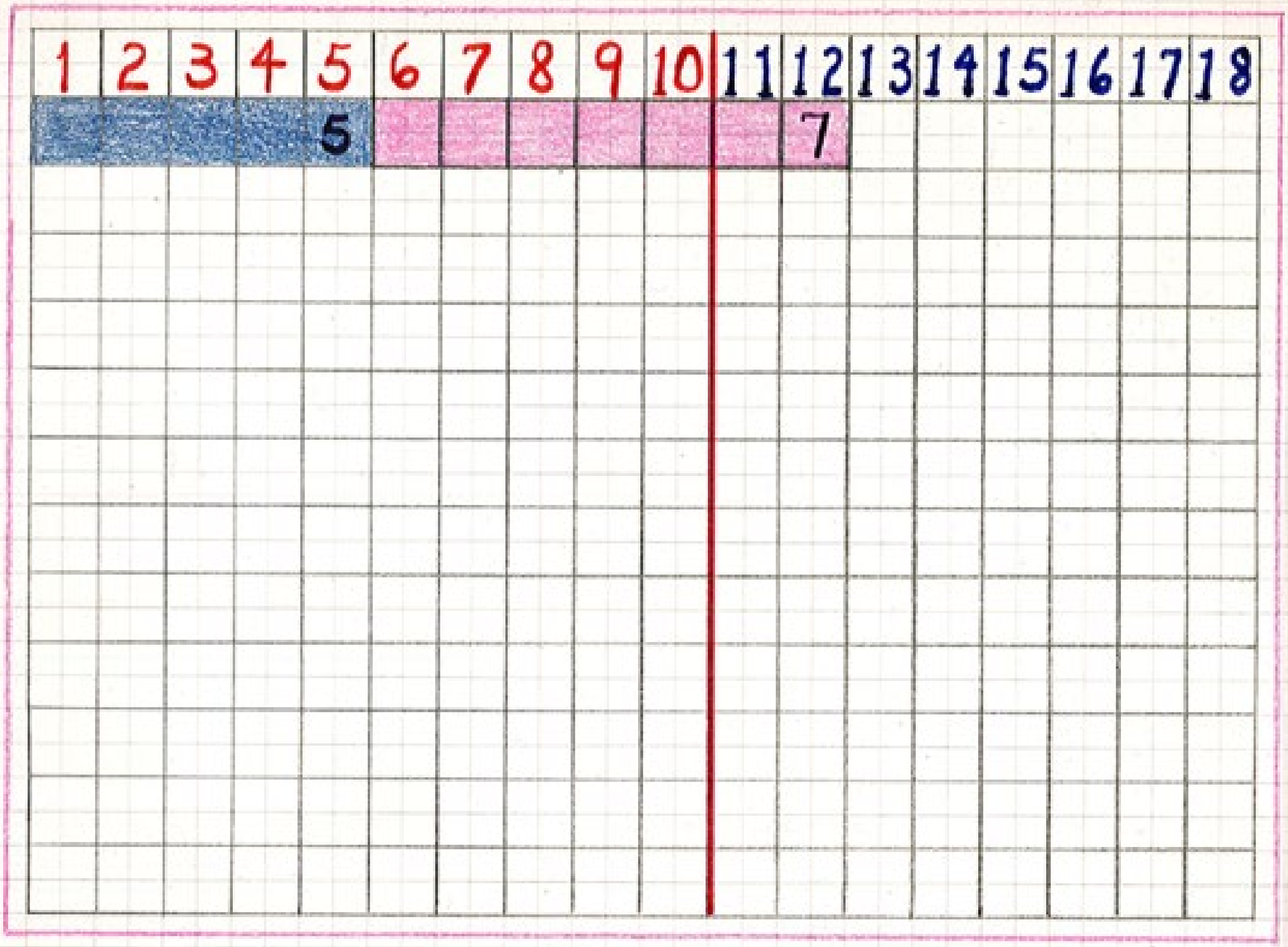
c. Since the next sum is 1 + 2 = , and so on through the 1s, he leaves the blue 1 in place and continues to change the pink strips in ascending order. Each time he reads the answer on the board and writes it.

d. When he has finished the 1s, it makes more interesting work to proceed to the threes. In this case the blue 3 remains throughout the page's combinations.

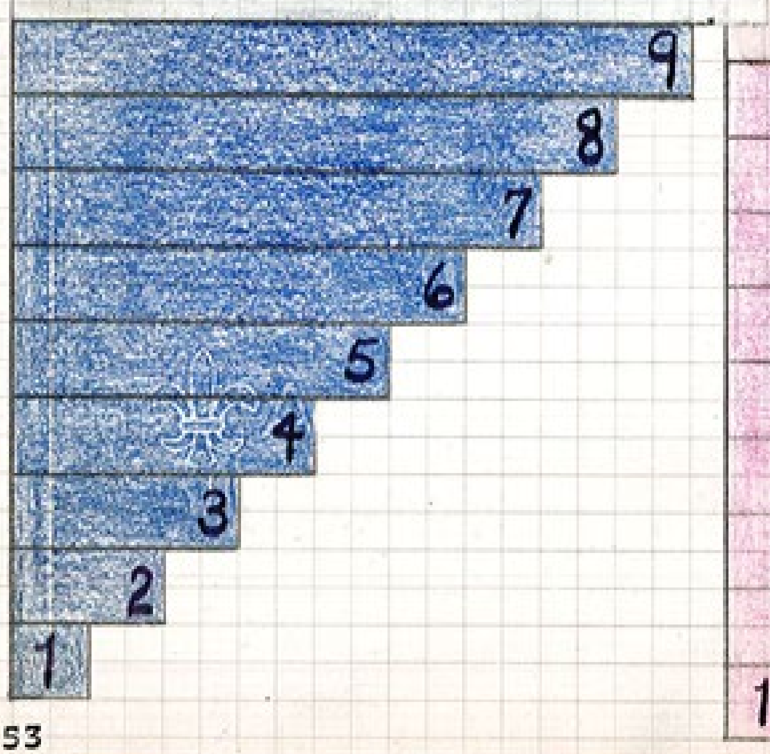
e. The child checks his work against Chart #1.

Addition Board

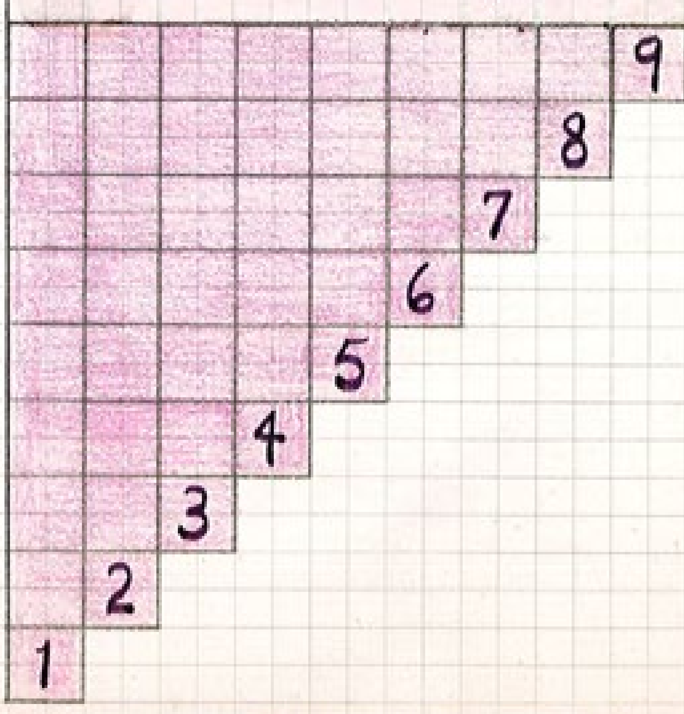
5 + 7 = 12



Blue wooden strips: first addent



Pink wooden strips: second addent



EXERCISE #1. . .

f. Teacher checks the child's work and has the child read the combinations on the chart that make 10.

f. Which are the combinations on our chart that make 10?
It's easy to find them because they are written in red, every 10.
Will you read them?

EXERCISE #2

Materials
1. Box of loose combinations in a box or bag.
2. Addition board and two sets of wooden strips.
3. Paper (or the child's notebook)
4. Chart #1

a. The child draws, out of the box or bag, one separate piece of paper.

b. On a separate piece of paper he writes the combination he has chosen.

c. With the addition board and the strips he makes the combination.

d. Then he writes the answer on his paper.

e. He draws another and repeats the combination.

f. The child checks his work with the first chart, looking for the combinations he has written.

g. He may discover that the similar results are in a diagonal on the chart.

EXERCISE #3: The combinations of numbers.

Material
1. The addition board and the strips.
2. CONTROL: Chart #1 and Chart #2.
3. Paper.

Presentation:

1. With the addition board and the strips, have the child discover all the different ways to make 10.

1. Let's see how many different ways we can make 10 on our board.
We know that we must always finish at the red line.

2. Help the child until he is able to continue by himself, asking him to write each of the combinations in his notebook.

2. 1 plus what makes 10?
1 + 9 = 10.
Will you write that in your notebook?

EXERCISE #3. . .

3. Review the combinations that the child has written.

3. 1 + 9 = 10
2 + 8 = 10
3 + 7 = 10
4 + 6 = 10
5 + 5 = 10
~~6 + 4 = 10~~
~~7 + 3 = 10~~
~~8 + 2 = 10~~
~~9 + 1 = 10~~

4. Show the proportional increase and decrease of the two addents.

4. We are finished.
What do you see?
We can observe that as the first blue quantity gets bigger, the second pink one gets smaller.

5. Note the sum 5 + 5.

5. See how the two fives are equal again as they were with the red and blue rods.
5 is half of 10.
Two 5s equal one 10.

6. Point out that there are really only five combinations. As the duplicate combinations are discovered, turn over the sticks on the board and have the child cross out: 9 + 1 = 10
8 + 2 = 10
7 + 3 = 10
6 + 4 = 10

6. Let's look at the combinations we have made on the board.
We can see that 9 + 1 gives the same result as 1 + 9.
So it is enough to learn 1 + 9 = 10 to remember that 9 + 1 = 10.
Let's turn over our strips to show that. . .and will you cross out that combination in your notebook?
Do you see another two combinations that are made with the same two quantities?

When we come to the two equal fives, nothing can be crossed out.

Now we have only five combinations left.
When you know 1 + 9, you know 9 + 1, etc.
It is enough to memorize only five combinations!!

Presentation: A Preparation for Using Chart II
Moving Towards All the Addition Combinations
Indirect Aim: To begin to understand the commutative law.

1. Examine Chart I with the child. Note diagonals of similar results and then analyze the combinations that produce similar results.	1. Look carefully at our first table. See how the totals that are the same can be found on a diagonal. Point to each of the 10s; they are easy because they are red. Now find each of the results which are 5. Let's look at the ways 5 is made.
2. Discover with the child the combinations it is necessary to memorize. Cover the duplicate combinations with pink slips of paper. (Child may write those combinations that are necessary to learn; or begin by writing them all and crossing the unnecessary ones out.)	2. Now let's look at the total 2. How many ways is 2 made? Then we must learn that: 1 + 1 = 2. Let's study the combinations that make 3. We have 1 + 2 and 2 + 1. We can cross out 2 + 1 because it is the same thing as 1 + 2. Once we have learned that 1 + 2 = 3, we know that 2 + 1 = 3.
3. Continue through all the numbers. Always equal addents have no duplicates. As the larger numbers are examined, the work becomes more interesting with several duplicate combinations for one result.	
4. When the duplicates have all been discovered, half of the chart is covered---all combinations to the right of the center diagonal. Explain that there are only those remaining (48) which are necessary to memorize.	4. We can see that it is not necessary to memorize all 81 combinations. We will take away 9, because we know we must learn those combinations with equal addents. We have to memorize half of the rest: 72 ÷ 2 = 36. We must learn then 36 + 9 = 48 combinations to know them all!!
5. Present the Chart II; note how it is formed and that all the combinations are still present.	5. To form this table we have used only those combinations which we did not cover up on Chart I. But here it has been rearranged so that all the 10 totals are lined up. We can still find the combinations we crossed out on this table.
6. Note that all tables are on the Chart II.	6. We can still find the whole table of every number. The table of 9 starts at the bottom and follows a diagonal. Can you find the table of 3?

ADDITION — Chart I

1+1=2	2+1=3	3+1=4	4+1=5	5+1=6	6+1=7	7+1=8	8+1=9	9+1=10
1+2=3	2+2=4	3+2=5	4+2=6	5+2=7	6+2=8	7+2=9	8+2=10	9+2=11
1+3=4	2+3=5	3+3=6	4+3=7	5+3=8	6+3=9	7+3=10	8+3=11	9+3=12
1+4=5	2+4=6	3+4=7	4+4=8	5+4=9	6+4=10	7+4=11	8+4=12	9+4=13
1+5=6	2+5=7	3+5=8	4+5=9	5+5=10	6+5=11	7+5=12	8+5=13	9+5=14
1+6=7	2+6=8	3+6=9	4+6=10	5+6=11	6+6=12	7+6=13	8+6=14	9+6=15
1+7=8	2+7=9	3+7=10	4+7=11	5+7=12	6+7=13	7+7=14	8+7=15	9+7=16
1+8=9	2+8=10	3+8=11	4+8=12	5+8=13	6+8=14	7+8=15	8+8=16	9+8=17
1+9=10	2+9=11	3+9=12	4+9=13	5+9=14	6+9=15	7+9=16	8+9=17	9+9=18

Addition — Chart II

1+1=2								
1+2=3								
1+3=4	2+2=4							
1+4=5	2+3=5							
1+5=6	2+4=6	3+3=6						
1+6=7	2+5=7	3+4=7						
1+7=8	2+6=8	3+5=8	4+4=8					
1+8=9	2+7=9	3+6=9	4+5=9					
1+9=10	2+8=10	3+7=10	4+6=10	5+5=10				
	2+9=11	3+8=11	4+7=11	5+6=11				
		3+9=12	4+8=12	5+7=12	6+6=12			
			4+9=13	5+8=13	6+7=13			
				5+9=14	6+8=14	7+7=14		
					6+9=15	7+8=15		
						7+9=16	8+8=16	
							8+9=17	
								9+9=18

EXERCISE #4: This is exercise #3 with the addition of a new element: An introduction of the zero.

Material

1. The addition board and the pink and blue strip stairs.
2. Paper.
3. Chart I or Chart II---CONTROL

Presentation

1. With the addition board and the strips the child discovers all the combinations that give 8. He writes each one in his notebook. BUT---he begins with

   0 + 8 = 8

   Teacher writes the combinations on the board.

1. Today we want to find out all the combinations that make 8.
   Let's start with 0 + 8.
   We know that zero plus a number does not make the total any more.
   How will we make that combination on our addition board?
   What are the other combinations that make 8?

2. The last combination will be 8 + 0 = 8.

2. Then 1 + 7 = 8
   2 + 6 = 8
   3 + 5 = 8
   4 + 4 = 8
   ~~5 + 3 = 8~~
   ~~6 + 2 = 8~~
   ~~7 + 1 = 8~~
   And our last combination will be: 8 + 0 = 8.

3. Note the solid color strips at both the top and the bottom. Point out that the zero does not change the total.

3. It is interesting that both the first and the last strip on our board is one color.
   Why is that?
   With 8 + 0, we really only have one addent.

4. Child looks for the duplicate combinations and crosses them out on his paper, turning them over on the board.

4. Can you discover how many different combinations there are?
   Which ones are the same?
   How many will you need to memorize?

5. Check with Chart I or Chart II.

EXERCISE #5: The double of numbers

Direct Aim: To memorize these combinations with equal addents.
Indirect Aim: To prepare for understanding Chart V.

Materials

1. Addition board and strips.
2. Paper or notebook.
3. Chart II - CONTROL

Presentation

1. Child makes the combinations with equal addents on the addition board with the strips. He reads the total on the board and writes the whole combination in his notebook. The teacher writes it on the board.

1. Today we will look carefully at the double of our numbers. We will begin with 1 + 1. What is the result? Now make 2 + 2 on the addition board. What is the total?

2. Show that none can be eliminated.

2. Can we eliminate some of these combinations? No, we need to learn all of them.

3. Check answers with Chart II.

3. We can check our totals on our Chart II. The combinations with the same addents are found on the top diagonal.

NOTE: Each of these combinations is made on the addition board, then removed before the next one is made.

EXERCISE #6: Utilizing Chart III on which there are no combinations, only the results. Thus in his progress towards memorization of the combinations, this represents a further abstraction.

Materials

1. Chart III.
2. The box of loose combinations.
3. Paper & pencil or the child's notebook.
4. CONTROL: Chart I.

Presentation

1. Child draws a combination out of the box and writes it in his notebook.

2. Show the child how to find the total on Chart III.

2. How do we look for the total? I find the first addent on the blue strip at the top and place my finger there. Then I find the second addent on the pink strip, and place a finger on it. Finally I slide this top finger down and the other one across until they meet. And there is the result!!

EXERCISE #6. . .

3. Child writes the result in his notebook and then draws another combination. Finds the answer in the described manner and reads it from the **chart**, then copying it.

4. Child checks his work with Chart I.

Presentation: A Preparation for Using Chart IV

1. **Examine Chart** III with the child, noting the diagonals again on which the equal results are found.

1. Look closely at this Chart III. Here we have **only** the totals of our combinations. But, like our first chart, the same totals are found on the diagonal. Draw your finger across the 6s. Now notice the numbers on each side of the 6s. They are the same. Now run your finger down the middle diagonal. There are the same number of numbers each side of it.

2. As with Chart I, half of the totals on Chart III can be eliminated. Now the child must THINK of the combinations which give him the totals. As he discovers the duplicates, cover each with a pink square. If he cannot remember the combinations, he can make them with his fingers on the chart.

2. Remember how we covered almost half of the combinations on Chart I? We cannot eliminate those totals made with equal addents. Half of the rest are duplicates. Let's cover those up.

3. When Chart III has been covered to the right of the middle diagonal explain how we move to Chart IV.

3. This is our fourth chart. On it we find only those totals which we need---the ones left after we covered all those that were the same as another one.

EXERCISE #7 : The Half Table

Material
1. Chart IV.
2. Loose combinations in box.
3. Pencil & paper or child's notebook.
4. Chart I---CONTROL

Addition — Chart III

0	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

Addition — Chart IV

1	2								
2	3	4							
3	4	5	6						
4	5	6	7	8					
5	6	7	8	9	10				
6	7	8	9	10	11	12			
7	8	9	10	11	12	13	14		
8	9	10	11	12	13	14	15	16	
9	10	11	12	13	14	15	16	17	18

EXERCISE #7. . .

Presentation

1. The child draws a combination from the box. Teacher shows him how to find the total on Chart IV. Child writes combination in his notebook.

1. Draw a combination from the box. What is it?
5 + 4 =

We must always take the smallest addent first when we use this board.
We know that 4 + 5 = 5 + 4.
So first, place your finger here on the pink 4 and slide it all the way to the end of the row.
Now place a finger on the 5 and move it across AND move the first finger down, stopping beside that second addent. That is the total.

2. Child reads the result and writes the result in his notebook.

3. Child draws another combination and repeats. Important to emphasize that the smallest addent must be found first so that the finger can move all the way to the end of the line before descending.

3. 9 + 3 =
9 + 3 is the same as 3 + 9.
Remember that whenever we have the biggest addent first, we must turn it around before we look for the result on this chart.

4. Child checks his work with Chart I.

EXERCISE #8 : The simplified Chart V

Direct Aim: To proceed with the memorization of the combinations using a higher level of abstraction in the material---and thus providing another step towards memorization.

Indirect Aim: To begin to show the child that: when the sum has 2 even addents, the result is even. 1 even and 1 odd addent result in an odd total. And adding 2 odd numbers results in an even total.
A further elaboration of even and odd numbers.

Material

1. The box of loose combinations.
2. Paper or the child's notebook.
3. Chart V and IV.
4. Chart I---CONTROL.

Presentation

1. Show Chart IV and Chart V on the table together and briefly describe the further reduction.

If the child is interested, he may copy Chart IV and then color in red those numbers which are not found on Chart V.

1. Look carefully at the chart you have been using and this new one---Chart V.
Let's cover the pink numbers.
Now we can see that on this chart each of our totals is only found one time.
Look at the top diagonal on Chart V.
What kind of numbers are they---even or odd?
And the second diagonal below?
This is a very interesting chart.

We can still find on Chart V the total for every combination.

2. Put Chart IV aside.
Child draws a combination from the box, and the teacher shows him how to find the total on Chart V.

2. 4 + 6 =
Watch how I find the total.
I place my finger on the first addent and follow its road to the end.
I place another finger on the second addent and follow the row to the end.
Now I move my fingers together on the top diagonal, <u>one square</u> at a time until they meet.
The square on which they meet is the total.

OR

2 + 7 = ...
Then my two fingers jump one square at a time towards each other, but
there is no square where they can meet.
They must drop down to the sqaure below. There is our result.

EXERCISE #8: Chart V. . .

3. The child repeats what the teacher has done, finds the total and writes it in his notebook.

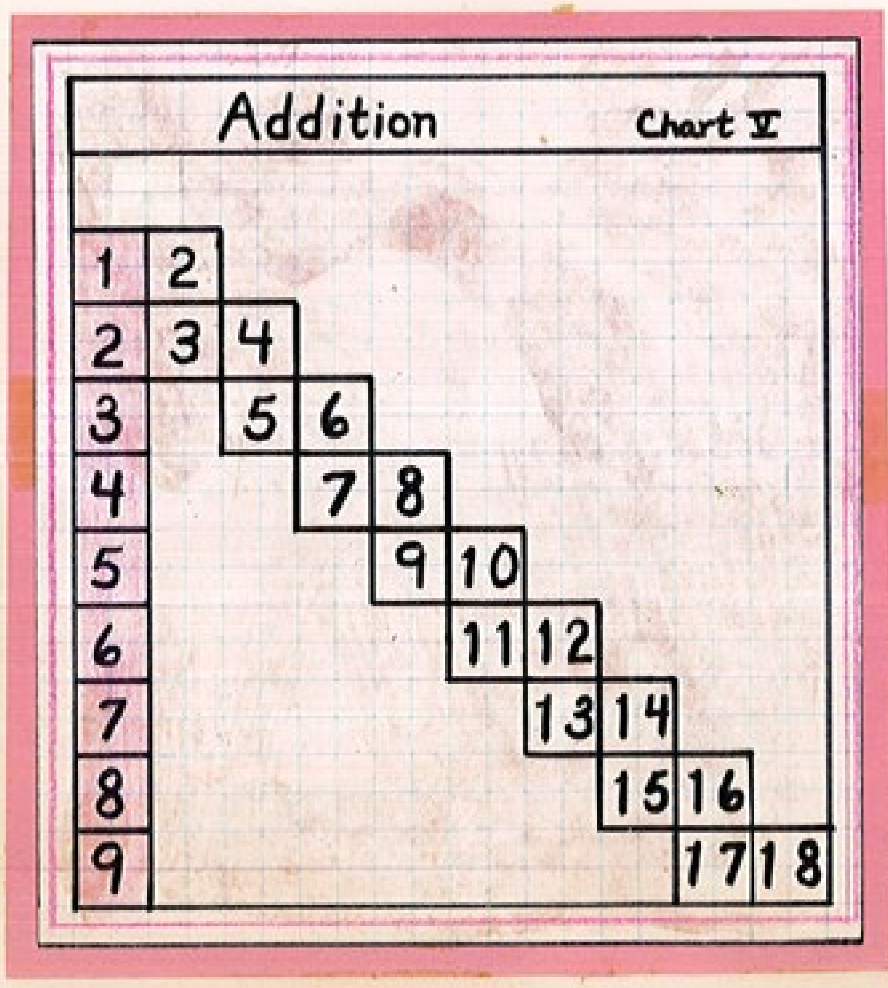
4. The child draws another combination and finds the total on the board. The teacher may have to help again on the second or third try.

5. Then the child continues the exercise, making a long list of the combinations and totals with the Chart V.

6. He checks his work with Chart I.

7. When he has a long list written, observe with him the combinations that give an even result and those that give an odd.

7. Let's look carefully at the totals you have written.
Show me an even total.
What kinds of numbers form that total---even or odd?
Show me another.
Show me an odd total.
What kinds of numbers form it?
We can see that when we have 2 even addents, the total is even.
When we have two odd addents, the total is even.
When we have one odd addent and one even, the total is odd.



The teacher's reference:

	first addent		second addent		TOTAL	
	EVEN	ODD	EVEN	ODD	EVEN	ODD
4+6=10	4		6		10	
7+2=9		7	2			9
3+7=10		3		7	10	
5+8=13		5	8			13

NOTE: Chart V, as well as III and IV are constructed on a progression of 1. The following formula, then may be superior for the explanation of Chart V:

Let x = the larger number and y = the smaller addent

$x = 7;\ y = 2$

Then: $2x - (x-y)$ and $2y + (x-y)$ are valid solutions.

$2x = 14 - (7 - 2) = 14 - 5 = 9$ and $2y = 4 + (7 - 2) = 4 + 5 = 9$

EXERCISE: For the child of 9 or 10: A Reference to Algebra

When this exercise is presented, the child has long since abandoned the addition memorization material including Chart V. Since he knows all the combinations now, he has probably forgotten the chart and how he used it.

Material

1. Chart V.
2. Paper, the child's notebook.

Presentation

1. Show Chart V. Introduce the term algebra.
   Algebra (Al, Arabian prefix; giobr, Medieval Latin) actually means restoration.
   It means then to reconstruct with symbols & signs.
   Classically, algebra means the set of rules of letteral calculus.

   1. Do you remember when we worked with this chart?
   How did we find the total for a combination of two addents?
   Now we are going to show what we did with this chart by the process of algebra.
   The word algebra means to reconstruct something with symbols and signs.

2. Now we explain the mechanism of the chart to the child by putting into different words the process he followed at a lower level.

   2. We are going to reconstruct what we did with this chart by using algebraic terms and symbols.

3. Say that we will call the first addent x.
   And the second addent y.

If $4 = x$

and $6 = y$

$(4+6) = (x+y)$

How did we find the total?
The last square on the row is the double of the number.

So $(4\times2)+(6\times2) = 2(x+y)$

and $4\times2+6\times2 = 2x+2y$

$20 = 2x+2y$

$20 = 2(x+y)$

When our fingers met in the middle, we divided by 2.

So $\frac{20}{2} = \frac{2(x+y)}{2}$

and $10 = x+y$

4. We can conclude that:
   On Chart V, the sum of two numbers is equal to the double of the two numbers divided by 2.

OR: $(x+y) = \frac{(2x+2y)}{2}$

EXERCISE #9: Addition Bingo Game

Direct Aim: To find out if the child has memorized the combinations.

Material

1. Box of loose combinations.
2. Box of wooden stamps in the following number for each of the numerals: one 2, two 3, three 4, four 5, five 6, six 7, seven 8, eight 9, nine 10, eight 11, seven 12, six 13, five 14, four 15, three 16, two 17, one 18. This group is the totals for all possible combinations. (81)
3. Chart VI---the last reduction. Here we have only the pink strip of numbers representing the first addent and the blue strip for the second.
4. Charts I and III---CONTROL
5. Paper, the child's notebook.

Game A

1. The child spreads out all the wooden stamps face up next to Chart VI.

2. Child draws a combination and writes it in his notebook.

3. Using his fingers and guiding from the blue and pink addents, he finds the proper square for the total.

4. He holds his place with one finger and finds the right stamp for the total. He covers that square with the total.

5. Child writes the answer in his notebook.

6. IF THE CHILD DOES NOT REMEMBER THE TOTAL:
   Uses Chart I to find the correct result for his combination.
   Uses Chart III to find the correct placement of the stamp.

7. Checks his work with Chart I and Chart III for correct total and placement.

Game B

1. Put all the stamps in the box. The child draws a stamp.

   1. First choose a stamp from the box. (12) What combination gives 12? Which one shall we use?

2. Child writes down the combination he decides to use for the number he has drawn.

3. Then he repeats the process in Game A, covering the proper square on the Chart VI.

4. He draws another stamp and continues.

Game C

1. Put all the wooden stamps on the mat and then arrange them in stacks, putting together each of the totals. Then make them into a graduated line with the ten totals in the middle stack and all others in decreasing order.

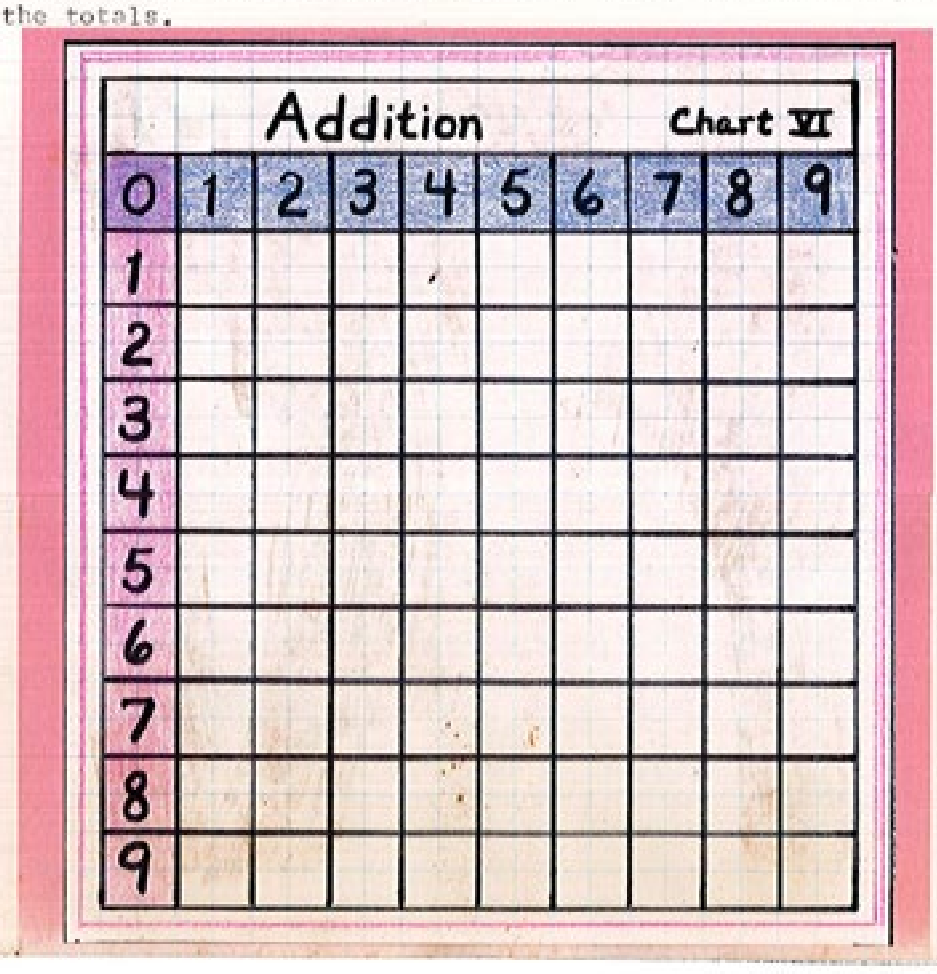
```
                    10
                  9 10 11
                8 9 10 11 12
              7 8 9 10 11 12 13
            6 7 8 9 10 11 12 13 14
          5 6 7 8 9 10 11 12 13 14 15
        4 5 6 7 8 9 10 11 12 13 14 15 16
      3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
    2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
```

2. Ask the child what form he sees. He may observe a triangle or a pyramid. He may also note that the most possible combinations for any total is 9---and that is the number of combinations which give 10.

3. Choose all the totals of one number and place them on Chart VI, finding the correct position by marking the various combinations.

4. Child will recognize as he places the totals that the equal totals are always on a diagonal.

5. CONTROL: Chart I for the combinations. Chart III for the position of the totals.



Game C. . .

Two Exercises: To see if the child has memorized the combinations.

1. The teacher gives a combination or the child draws one from the box of loose combinations. Then he takes the stamp for the correct total and places it on Chart VI.

2. The child draws a stamp and the teacher asks for all the puzzle combinations that give that total.

AGE: For all memorization exercises: 6, 7 years. (excepting algebra)

SPECIAL ACTIVITIES

Direct Aim: To maintain the child's interest in addition. To provide a review of all the exercises for memorizing the combinations and to reinforce that memorization on a new level of abstraction. To help the child reason in addition.

Indirect Aim: A preparation for the operation of subtraction.

NOTE: We now introduce a series of 6 special combinations:

1. How to find the second addent: 6 + ? = 8.

2. How to calculate the first addent: ? + 2 = 8.

3. Knowing the total, to find the second addent:
   8 = 6 + ?

4. Knowing the total, to calculate the first addent:
   8 = 2 + ?

5. To calculate the total: ? = 6 + 2.

6. To calculate both of the addents: 8 = ? + ?

Materials

1. The teacher prepares in a box cards on which are written several examples of each of these special cases.
2. In another box cards on which are written simple verbal problems which demand a solution based on one of the six cases:
   Es. How many pieces of candy will you have if John gives you 3 and Robert gives you 6 caramels?

Presentation #1: Special combinations.

1. The child draws from the box one of the combinations. He must make a mental addition. Writes the combination and completes it in his notebook.

1. 6 + ? = 8
   6 plus how many gives 8?

   8 = 6 + ?
   Eight is equal to 6 plus how many.

   ? = 6 + 2
   What would the total be if I added 6 and 2.

SPECIAL ACTIVITIES. . .

Presentation #1. . .

NOTE: The special cases 1-4 can be made through a subtraction, but it is important that the child solve the problem through a mental addition.
Cases 5 & 6 cannot be obtained through subtraction.

2. Child uses Chart I if he does not know the answer.

Presentation #2: Verbal expressions of the special cases

1. The child draws from the box a card on which is written a verbal addition problem. If he has understood the previous problems, he will know now to put it into arithmetical form.

1. John has 8 cars. If 6 of them are new, how many are old?

How can we write this as an addition?
8 = 6 + ?

2. Child writes first the verbal problem in his notebook, then the arithmetical solution--- or he may want only to write the arithmetical expression. Then he writes both the complete arithmetical solution and the verbal answer.

2. How shall we write our answer?
8 = 6 + 2 AND
John has 2 old cars.

NOTE: It is important that the child knows how to read before he is able to use this exercise. It is a most important one for understanding arithmetical functions---and should be used often at this point in his progress.

MEMORIZATION GAMES FOR ADDITION

Direct Aim: To help the child in his work of memorizing all the addition combinations.

THE SERPENT REVISITED

Materials

1. The materials used in the serpent game: the divided box containing the colored bead stair, the box of golden tens, the black & white stair.
2. Chart I.

Presentation

1. Teacher forms a snake of the colored bead bars and lays out the black & white stair.	1. Take the first two bars from the snake and place them below.
2. By now the child knows the bars by color, so he can read the combination without counting the beads.	2. What two quantities have we put together? (The child says 8 + 8)
3. The child takes the golden ten and the B&W stair bar that he needs to represent his total. IF HE DOES NOT KNOW THE TOTAL, he consults Chart I.	3. What is the total? You can find it on Chart I. Take, then, the golden ten that you need and which of the black and white beads? (6)
4. Child replaces the two colored bars in the box lid and the new ten and B&W bar on the snake.	
5. Then he brings away from the snake the B&W bar and the next colored bar, reads the combination, and repeats the process.	5. Now bring away from the snake your black & white bar and the next colored bar. What are we adding together? What is the total?
6. When the snake has become golden tens, the quantity remaining less than ten is replaced by the B&W bar.	
7. The child then makes the proof, taking the colored beads from the lid, placing them in order from the longest; matching them with the 10s and exchanging when necessary to complete the match.	

GAME #1: Addition without carrying over

Direct Aim: To show that an addition can be made with bead quantities.

Material

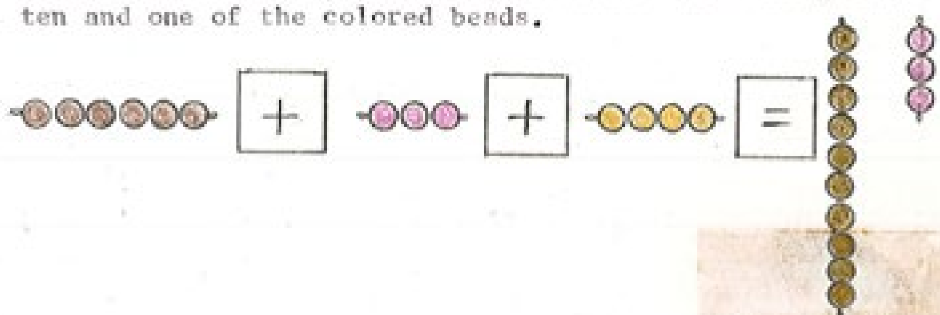
1. A box of colored bead bars which is divided into ten sections so that the quantities from 1-10 are represented in the bead bars.
2. The box of operation symbols: + = - ( ) X _______, etc.
3. Long strips of paper.
4. Chart I

Presentation A : Total Less than 10.

1. The teacher writes an addition combination on a slip of paper and asks the child to write it in his notebook. . .
   OR BEGIN WITH STEP 2 TO SIMPLY SHOW HOW BEADS ARE USED:
2. Then she shows the child how to make the combination with the bead bars and the symbols of operation.
3. Child then places the correct bead bar for the total.
4. IF THE CHILD DOES NOT KNOW THE COMBINATION'S TOTAL, HE DOES NOT COUNT THE BEADS. He consults Chart I.
5. The teacher uses combinations of two AND THREE addents.

Presentation B : Total More than 10

1. Repeat as in A, only the teacher writes a combination that totals more than 10, for which the child uses the golden ten and one of the colored beads.



GAME #2 : Addition without carrying over: The Commutative Law

Materials (-same as game #1-)

Direct Aim: To show the commutative law.

Presentation

1. The teacher prepares on a paper slip two addition combinations, one the inverse of the other. Es: 9 + 8 = and 8 + 9 =
2. The child forms each with the bead bars, one below the other and lays out the two equal totals. Consults Chart I if necessary.
3. Gradually the child recognizes that changing the order of the addents does not change the total. Do not give the rule; let him discover it through a series of these problems.

GAME #3: Addition without carrying over: The Associative Law

Direct Aim: To introduce the associative law.
To introduce mental division where the child must keep in mind the total of one combination and add the next---beyond Chart I.

Materials (the same)

Presentation

1. The teacher writes an addition on a paper slip with several addents. This is an advanced level, going beyond the Chart I combinations.
Es: 4 + 6 + 2 + 5 =

2. The child lays out the corresponding beads, and the total. Then writes it in his notebook.

3. When the child has done many of these exercises, have him replace two of the addents with an equivalent bar.

3. Let's do something new. Remove the first two bars in your addition---the 4 & 6 And replace them with a 10. Now add the quantities together again.

4. Refer to the associative law.

4. It is interesting that the total is the same.

NOTE: If the child has difficulty adding the 12 and 5---suggest that he add the units together first and then the ten.

GAME #4: Addition without carrying over: Parentheses

Direct Aim: To introduce work with parentheses

Materials (the same)

Presentation

1. Immediately following the above exercise of changing two bars for an equivalent, show the use of parentheses.

1. These are the signs we use to show what we just did: ( ) They are called parentheses. When we find these parentheses around certain numbers in our problem, it means we must first solve the addition WITHIN them and then we add as usual.

2. Teacher writes on a paper slip an addition combination containing parentheses. The child makes the combination with beads and symbols.

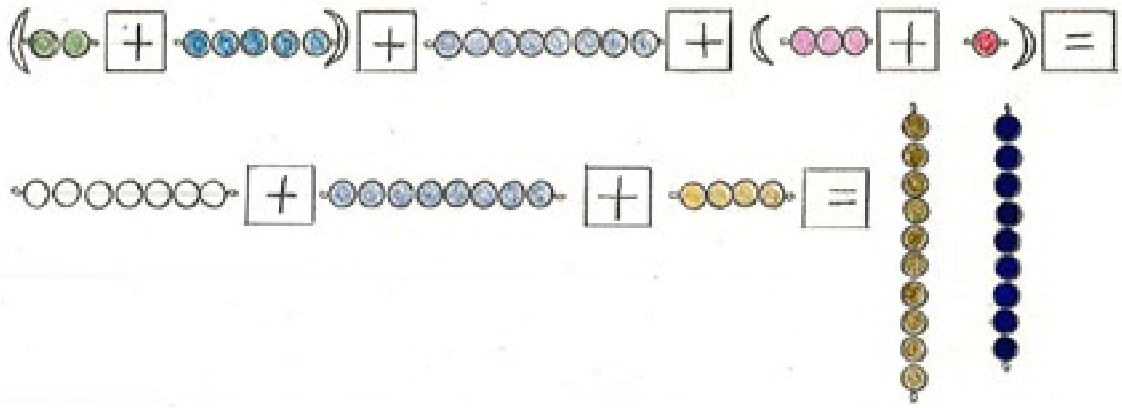
2. (5 + 2) + 8 + 3 =
Make this combination with the beads and the symbols.

3. The child solves the addition, replacing the sum in parentheses with the equivalent bar.

3. Remember that you must first solve the addition in the parentheses, taking away the bars and the symbols and replacing there the equivalent quantity.

GAME #5: Addition without carrying over: Using parentheses

The teacher writes an addition with more than one set of parentheses and the child lays out the addition with the beads and the symbols. Then he shows the addition, having solved the parentheses and makes the total.



GAME #6: Addition without carrying over: The associative law

Indirect Aim: To give the associative law. A subconscious preparation.

Materials (the same)

Presentation

1. Teacher writes a combination with three addents and child makes it with beads, finding the total.

   1. 9 + 7 + 6 =

   ●●●●●●●●● + ooooooo + ●●●●●● =

2. Teacher prepares the combination with each of the addents shown as two numbers. Child makes this with beads and symbols below the first.

   2. (4 + 5) + (3 + 4) + (5 + 1) =

   (oooo + ●●●●●) + (ooo + oooo) +

   (●●●●● + ●) =

3. Child simplifies his addition and shows the simplified addition below.

   3. ●●●●●●●●● + ooooooo + ●●●●●● =

4. The child observes as he removes the second combination that we have the same problem at the top and as a result of the second. He is seeing that a quantity can be broken down into two addents giving that quantity without changing the total. And that two such addents put together to form the quantity do not change the total.

GAME #7: Problems with addents biggen than 10: with carrying over

Direct Aim: To help the child abstract the process of carrying over.

Materials (the same)

Presentation A: Addents bigger than 10; no carrying over

1. Teacher writes addition on slip: 12 + 14 =

2. Child makes the addition with the beads and symbols.

3. He is asked to add the UNITS FIRST. Then the tens. Makes total.

Presentation B: Addents bigger than 10; carrying over

1. Teacher prepares the addition on paper slip. Child makes it with beads and symbols.

   1. 18 + 25 =

2. Show the child how to add the units first, put down only the unit part of the total, hold the ten, keeping it in mind and adding it to the tens.

   2. How many units do we have? 5 and 8 make 13. I put down the 3, but this is a 10. I will hold it to keep this 10 in mind. I have 1, 2, 3 tens with this 10 makes 40. We have 40.

SECOND TIME:

1. Teacher prepares the addition and child makes it with beads and symbols.

   1. 16 + 37 =

2. This time show the child how to carry over the ten without ever picking up the bar.

   2. We have 13 when we add 6 units and 7 units. I put the 3 down to show the units. Now we are not going to take the 10. We will just keep it in mind. The 10 we are keeping in mind is the one we say we are carrying over. So we have 4 tens and 1 more. We have 50.

GAME #8: THE DOT BOARD

Direct Aim: To help the child understand that 10 units of a lower order form one unit of the next higher order.
POINT OF CONSSCIOUSNESS: To make the child understand carrying over.

Material

1. The dot board, a wooden frame in which the chart (pictured on the opposite page) is fitted and covered with rough glass. On the glass the child can write in pencil and red colored pencil and then rub the marks out for another problem.
2. If the board is not available, printed sheets with the chart may be used; and are, in fact, good to have in addition since the children enjoy making the solution with the dots and then keeping their work.
2. Black and red pencil.

Presentation

1. Write on a slip of paper the quantity which is to be added. The child copies it on the far right column of white.

2. Child first adds the units on the board by making a dot under the units column for every unit. Continues through the hierarchies.

2. How many dots shall you mark for the units? We fill the first row of ten squares under the units and then begin the next when we need it.

3. Show the child how to make the operation after all the dots have been made.

3. First we cancel each row of ten dots that is filled in our unit column. Put a red dot in the first white space below for every row canceled.

Count the red dots at the bottom and put the number just to the left of the line under the 10s. Then put that number of red dots in a row under the 10s. Those are the 10s we have carried over. Now count the remaining dots in the unit column. Write the number below in the bottom white space.

4. Proceed through the 10s, 100s, 1000s, finally 10,000s.

10,000	1000	100	10	1	
					4312+ 784+ 5622+ 3189=
	1	2	1		
1	3	9	0	7	

THE MEMORIZATION OF SUBTRACTION

Direct Aim: To teach the child to memorize all the possible subtraction combinations within the limits of 18. The combinations are formed by two numbers---one minuend and one subtrahend. THE RULE is that the maximum difference cannot be greater than 9.

Material

1. The subtraction board. Numbers 1-9 at the top in black, numbers 10-18 in red; a blue line divides the board after 9.
2. Three sets of wooden strips: blue strips 1-9
   pink strips 1-9 with unit divisions
   neutral strips, unnumbered (1-17)
3. A light blue box of loose combinations. (those cut into separate slips from the subtraction booklets.)
4. Booklets of the printed combinations: a page for each number from 1-18 and beginning with 18 on which is found one combination:
   18 - 9 = 9.
5. A box of wooden stamps, these blue and representing the differences.
6. Charts I, II, III

Presentation: To familiarize the child with the subtraction board and the use of the strips.

1. Place on the mat the subtraction board and lay out the blue strip stair and the neutral strip stair. (leave pink in box)

1. This looks like the addition board, but there is a difference. Here the line is between the 9 and the 10.
It comes after the 9 because THE RULE OF THE GAME IS THAT THE DIFFERENCE CANNOT BE BIGGER THAN 9.

The numerals at the top show us the minuend, the number <u>from</u> which we will subtract another number. We use these netral wooden strips to make the minuend we need.

These blue strips make our subtrahend, the number we take away.

2. Demonstrate the subtraction mechanism of the board.

2. Let's see how we can subtract with this board.
Let's subtract 5 from 13.
First I cover the numerals at the top, from the highest to the numeral 14.
Now I can see the minuend of 13.
Next I place the blue strip 5 next to the first strip.
And where the blue strip ends I find the numeral that is the difference---8.

3. Repeat several times until the child knows it well. Note that no neutral bar is needed for a minuend of 18.

3. If I want to subtract 18-9, I don't need to cover any of the numerals at the top.
My minuend is 18---our last numeral.
Take the blue stick 9 as the subtrahend.
Now read the answer.

EXERCISE #1

Material

1. The subtraction board.
2. The blue strip stair and the neutral stair.
3. The subtraction booklet of printed forms.
4. Chart I.

Presentation

1. Have the child begin his work with the first page of the subtraction booklet: 18. Note that there is only one combination because of the rule of the game.

1. We can use our subtraction board to find the difference for every combination in this booklet.
Here we have 18-9 = 9.
Why is there only one combination on this page?
The number of possibilities is limited by the rule which says that we can have a difference no bigger than 9.

2. The child finds the answer for the combination on the board, reads it and writes the answer in the booklet.

2. The minuend is 18.
You don't need a neutral stick because you need all of these numbers to subtract from.
Place the blue 9 on the numbers beginning at the 18.
Read the number you see before the strip.
What is the difference?
You may write it in the booklet.

3. Proceed to page 17.

3. We can go on to 17.
How many combinations are here?
How many numbers must I cover to make the minuend 17.
We can leave this neutral strip that covers just the one number for both the combinations on this page because we have the same minuend.

4. Each page has progressively more combinations per page.
If the child asks why we do not have a combination such as 16 - 6 = explain. . .

4. We do not have 16 - 6 = because we are not interested in a difference of more than 9.
If we had 16 - 6, we would first be subtracting 6 - 6, and that is on another page.
When we know these, we will know the others.

5. Child proceeds through the booklet, finding the differences on the board.

6. He checks his work with Chart I.

6. On Chart I we can find the differences for all the combinations in the booklet.
In each vertical row are the differences on one page.
What color are the differences?

NOTE: On the left half of Chart I we find the equal differences on a horizontal line showing the invariant property: that is, if we subtract, or add the same quantity to the minuend and the subtrahend, the difference doesn't change.
Thus: 2 - 1 = 1, 3 - 2 = 1, 4 - 3 = 1, 5 - 4 = 1
It is also interesting on the left half of the chart that we have all the combinations with a difference of 0.

On the right half of the chart only the minuend increases and the subtrahend is the same on each vertical row---so that the difference increases in a progression of 1. AND we get diagonals of similar differences.

EXERCISE #2

Material
1. The subtraction board.
2. The blue strips, the neutral strips.
3. The blue box of loose combinations.

Presentation

1. The child draws a combination from the box and writes it in his notebook.

2. Child works the problem with the strips on the board.

3. Reads the answer and writes it in his hotebook.

4. Then he takes the strips off and draws another combination.

5. Child controls his work with Chart I.

Subtraction

18

18 − 9 =

Subtraction

17

17 − 9 =

17 − 8 =

Subtraction Chart I

1-1=0	2-2=0	3-3=0	4-4=0	5-5=0	6-6=0	7-7=0	8-8=0	9-9=0	10-9=1	11-9=2	12-9=3	13-9=4	14-9=5	15-9=6	16-9=7	17-9=8	18-9=9
	2-1=1	3-2=1	4-3=1	5-4=1	6-5=1	7-6=1	8-7=1	9-8=1	10-8=2	11-8=3	12-8=4	13-8=5	14-8=6	15-8=7	16-8=8	17-8=9	
		3-1=2	4-2=2	5-3=2	6-4=2	7-5=2	8-6=2	9-7=2	10-7=3	11-7=4	12-7=5	13-7=6	14-7=7	15-7=8	16-7=9		
			4-1=3	5-2=3	6-3=3	7-4=3	8-5=3	9-6=3	10-6=4	11-6=5	12-6=6	13-6=7	14-6=8	15-6=9			
				5-1=4	6-2=4	7-3=4	8-4=4	9-5=4	10-5=5	11-5=6	12-5=7	13-5=8	14-5=9				
					6-1=5	7-2=5	8-3=5	9-4=5	10-4=6	11-4=7	12-4=8	13-4=9					
						7-1=6	8-2=6	9-3=6	10-3=7	11-3=8	12-3=9						
							8-1=7	9-2=7	10-2=8	11-2=9							
								9-1=8	10-1=9								

EXERCISE #3: The decomposition of numbers.

Materials

1. The subtraction board.
2. The blue strips, the neutral strips AND the pink strips, to show the difference.
3. Chart I.

Presentation

1. Show the different ways that the number 9 can be decomposed, using on the board the neutral 9 to show the minuend 9. Then the blue strips to show the subtrahend increasing from 1 - 8. And in each case, show the decreasing difference from 8 - 1 with the pink strips.

1. Let's find out how many ways I can decompose the 9.
First we must cover all the numbers back to 9 to show our minuend.
Let's begin with the blue strip 1, and this time we will place it on the first line of squares below the number.
What is the difference?
Let's use the pink strip 8 to show that difference.
So we see that 9 can be decomposed into two parts: 1 and 8.

2. Leave all the strips on the board as the exercise progresses. The child writes each of the combinations in his notebook.

2. 9 - 1 = 8
9 - 2 = 7
9 - 3 = 6
9 - 4 = 5 . . . . .

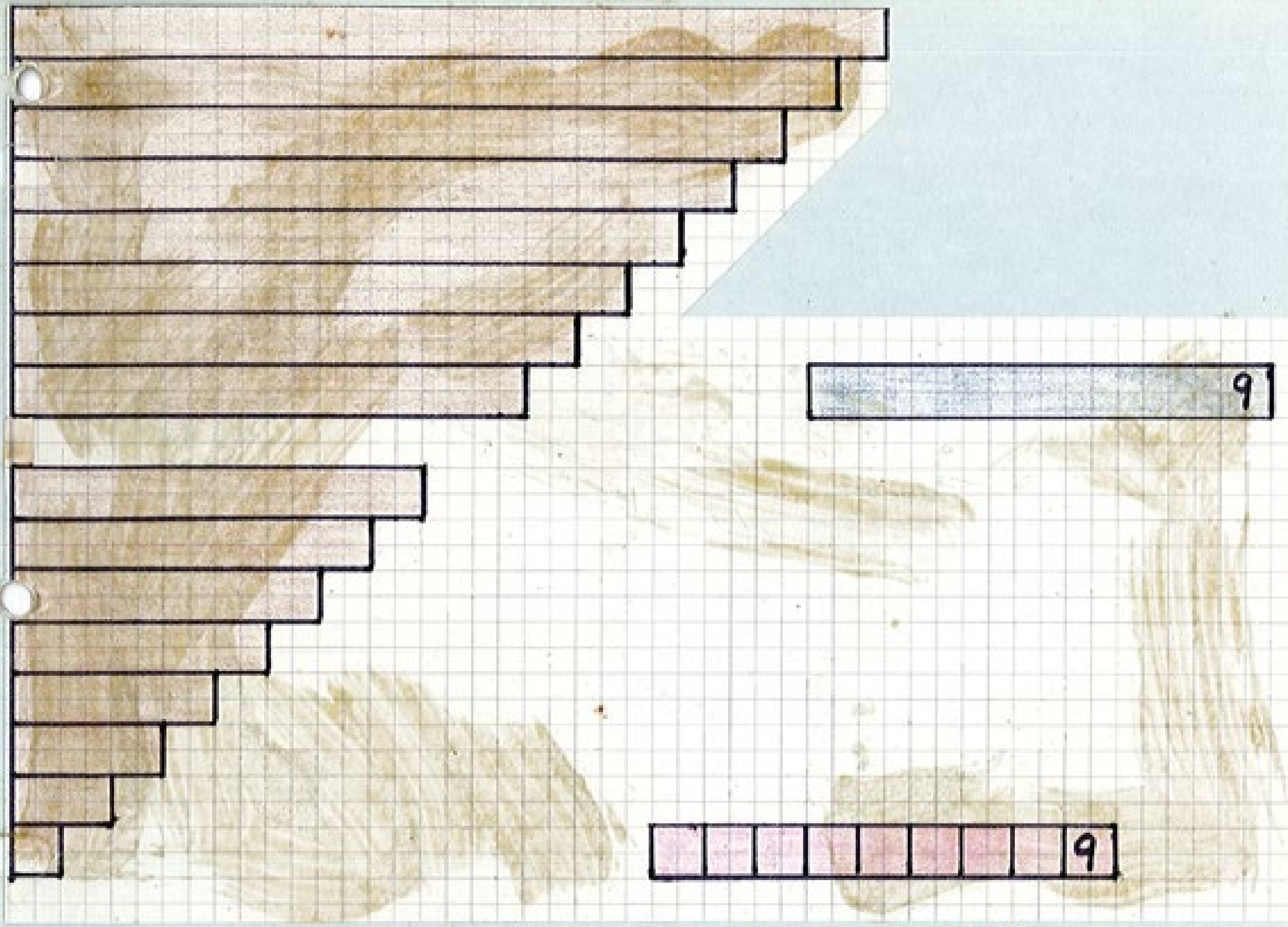
3. Note that as the subtrahend increases, the difference decreases. A VISUAL IMPRESSION.

3. As our subtahend gets bigger, what happens to the difference?

4. Note that none of the combinations is a duplicate.

4. Remember when we did this with our strips in addition.
We were able to cross out nearly half of our combinations.
Can we do this here?
Why not?
There are no duplicates----we cannot do the same thing in subtraction that we did in addition because in subtraction the order of the numbers cannot be changed.

5. Suggest to the child that he do this same exercise of decomposition with another number. If he chooses a number such as 14, remind him that we do not start with a subtrahend of 1, 2, 3, or 4 because 14 - 5 gives us the maximum difference of 9.



Subtraction

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
							8	1									
						7		2									
					6			3									
				5				4									
			4					5									
		3						6									
	2							7									
1								8									

MEMORIZATION OF SUBTRACTION. . .

EXERCISE #4: The Introduction of 0

Using the same materials and presentation as EXERCISE #3, proceed with the minuend of 7. And use as the first subtrahend NO blue strip, showing the combination 7 - 0. Then use the maximum difference---the pink 7. Continue through the decomposition of the 7, finishing with the maximum subtrahend 7: 7 - 7 = 0.

CONTROL: Chart I

Preparation: for the subtraction special combinations.

EXERCISE #5

Materials
1. Chart II.
2. The box of loose combinations.
3. CONTROL: Chart I.

Presentation

1. Introduce Chart II.	1. On Chart II, the minuend is shown in pink and the subtrahend in blue. In the white spaces are the differences. Notice that the equal differences are on a diagonal. How many possible combinations did we have in addition? (81) Here we have 90 because we have the nine combinations which give zero.
2. The child draws a loose combination and writes it in his notebook. Then he finds the difference on the Chart II with two fingers tracing the line and row.	2. 12 - 8 = I place one finger on the 12 and another on the 8. Then I move the first one down and the other one across until they meet. That is the difference --- 4.
3. Child writes the difference in his notebook and continues with another loose combination.	
4. Checks his work with Chart I.	

Subtraction																		Chart II
	18	17	16	15	14	13	12	11	10	9								
-9	9	8	7	6	5	4	3	2	1	0	8							
	-8	9	8	7	6	5	4	3	2	1	0	7						
		-7	9	8	7	6	5	4	3	2	1	0	6					
			-6	9	8	7	6	5	4	3	2	1	0	5				
				-5	9	8	7	6	5	4	3	2	1	0	4			
					-4	9	8	7	6	5	4	3	2	1	0	3		
						-3	9	8	7	6	5	4	3	2	1	0	2	
							-2	9	8	7	6	5	4	3	2	1	0	1
								-1	9	8	7	6	5	4	3	2	1	0

Following the rule of our game which says that no difference may be greater than 9; and then eliminating the possibility of negative numbers, the subtraction square below becomes the parallelogram of Subtraction Chart II.

	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
-9	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8
-8	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7
-7	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6
-6	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5
-5	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4
-4	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3
-3	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2
-2	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1
-1	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

MEMORIZATION OF SUBTRACTION

Direct Aim: The following three exercises are an indication of how well the child has memorized the subtraction combinations.

NOTE: These exercises utilize Chart III, a subtraction chart identical to Chart II except that the white squares are blank: that is, the differences are no longer written on the chart, but must be remembered from the combination of minuend minus subtrahend.

EXERCISE A

Materials

1. The box of loose combinations.
2. Chart III.
3. A box of blue wooden stamps representing all the possible differences, one for each of the combinations---that is, 90.
4. Paper, the child's notebook.
5. CONTROL - Chart I and Chart II.

Presentation

1. The child takes all the wooden stamps out of the box and lays them on the mat.

2. He draws a loose combination and writes it in his notebook.

3. Then he chooses the correct difference and places it on the square on Chart III that corresponds to the combination he has drawn.

4. He writes the answer in his notebook.

5. Continues with as many combinations as he likes and repeats the process.

6. If the child does not know the difference for the combination he chooses, he consults Chart I. If he does not know the correct position on Chart III, he consults Chart II.

8. The CONTROL is Chart I---for the correct results and Chart II---for the correct placement on the Chart III.

EXERCISE B

Materials (the same WITHOUT the loose combinations)

Presentation

1. The child puts all the stamps back in the box.

2. He draws one stamp from the box. (5)

3. Now he must write a combination that he chooses which gives the difference of 5. (Five is the difference of seven minus two)
$5 = 7 - 2$

EXERCISE B. . .

4. The child writes the combination in his notebook and places the stamp correctly on the square of Chart III which represents the combination he has chosen.

5. CONTROLS his work with Chart I and Chart II.

EXERCISE C

Material (same)

Presentation

1. The child takes all the stamps out of the box and puts the equal differences into stacks.

1. Remember how the stamps in our addition work made a pyramid?
How are our stacks different here?
There are an equal number of each difference.

2. The child chooses all the differences of one number and thinks of the combinations which give that difference. For each combination he writes in his notebook, he places one of the stamps on that combination position on Chart III.

2. Let's take all of our 3s.
How many do we have?
Which are the combinations that give 3?
3 = 9 - 6
So we can place one 3 on our chart to show that combination.
What is another combination that gives 3?
3 = 7 - 4

3. Proceed until all the equal differences of one number have been placed and note the diagonal which they form.

3. How many combinations do we have on the chart that give 3?
It is interesting that they fall on a diagonal.
Let's try another number.

ORAL EXERCISES: A child with a group or the teacher with a group of children play a game with the combinations. Reading the combinations from the loose combination box and the group giving quick responses. OR a wooden stamp is drawn, the difference number read, and the group answers with an appropriate combination.

Special Cases

Again in subtraction there are six special cases:

1. To calculate the subtrahend: 11 - ? = 4.
2. To calculate the minuend: ? - 5 = 7.
3. The difference :unknown: ? = 14 - ?
4. The subtrahend unknown: 5 = 14 - ?
5. Minuend unknown 9 = ? - 5
6. Minuend and subtrahend: 7 = ? - ?

MEMORIZATION OF SUBTRACTION. . .

Special Cases. . .

NOTE: In case 3, we have a typical subtraction BUT it is reversed. So that, in verbal problems, this case is obvious when the difference is mentioned first and then the minuend and the subtrahend given.
In case 6, we have the most difficult calculation for there are two unknowns.
The six cases represent a progression of increasing difficulty.

IT IS IMPORTANT TO READ THESE SPECIAL PROBLEMS CORRECTLY:

1. 11 minus how many gives 4?
2. How many minus 5 gives 7 as a difference?
3. What will the difference be if I take 5 from 14?
4. 5 is equal to 14 minus how many?
5. 9 is equal to how many minus 6?
6. 7 is equal to how many minus how many?

EXERCISE: The Special Cases

Direct Aim: To find out if the child has understood the arrangement of the special cases.

Material
1. A series of cards on which are written problems involving each of the special cases in subtraction. Es: ? = 9 - 3, 14 - ? = 7.
2. The similar series of problem cards for the special cases of addition.

Presentation

1. The teacher mixes the special cases for addition and subtraction.

2. The child draws one, writes it in his notebook, and then writes the unknown in red.

EXERCISE: Verbal problems in the special cases of subtraction

Material
1. A series of cards on which are written verbal problems involving one of the six special cases in subtraction.
Es: James had 9 trading cards left. How many trading cards did he have before if he lost 2 of them playing with Lewis. (Case #5: 9 = ? - 2)

Presentation: The child takes a card and may either copy it in his notebook or simply write the special case out as a numerical combination. Then he solves it, writing the unknown in red. 9 = 11 - 2

GAMES AND EXERCISES FOR THE MEMORIZATION OF SUBTRACTION

THE NEGATIVE SNAKE GAME

Materials

1. The box of colored bead bars.
2. Box of golden tens.
3. The box containing the black & white stair.
4. A box of grey bead bars representing the quantities 1-9. On the grey bead bars beyond five, there is a small space after the first five.

5. The box of operational signs.
6. Strips of paper.

Presentation

1. The teacher makes a colored bead snake. Lays out B&W stair.

   1. Today we are going to add something new to this snake.

2. Present the grey beads, first in a stair and then replace in box.

   2. Look at these bars. What color are they? They are not as beautiful as the others, but with them, we can play a special game. What is this? (1, 2, 3, 4, 5) This is 6. How is is different?

3. Explain the function of the grey bars and then include some in the snake, being sure to add them after several large quantities.
   Close both the boxes: grey beads & colored beads.

   3. Each time we find a grey bar in the snake, we take that quantity away. Let's include some in this snake. We have to put them after several colored bars so that we will have quantities first from which to take them away.

4. Begin counting the snake as usual, substituting golden tens and the B&W stair beads for the colored beads.

THE NEGATIVE SNAKE GAME. . .

5. When the first grey bar is reached, explain clearly how the subtraction is achieved.

Be sure the child sees that the 10 used for the subtraction goes back into the box of 10s.

5. Now we have a 7 and a grey 8. We can't take the 8 away from 7, so we must take our last 10 and add it to the 7.
Now we have 17 - 8.
That gives us 9.
We need the 9 from the B&W stair.
We put the grey bead bar into the box with the colored beads.
But we put the 10 back into the ten box because it has been used.

6. Note as these subtractions are done that the snake is growing shorter.

6. Look what has happened to our long snake.
It is becoming very short.

7. Make the check.
Line up the colored bead stair.
And the grey stair.

Then lay out the golden tens vertically and the remainder B&W.

8. Get rid of the grey bars by matching them with colored bars and putting them away.

8. To check our work, we must first make these grey bars disappear.
Here is a 9 and a grey 9.
We take away the grey 9 from the 9, and what do we have?
Zero.
Then we can put them away.

9. Now the remaining colored bars are matched with the tens as in the original snake game check.

9. Now we can check our snake.
There are no more grey beads---they don't count anymore.