*****

Multiplication Chart III

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Multiplication Chart IV

1									
2	4								
3	6	9							
4	8	12	16						
5	10	15	20	25					
6	12	18	24	30	36				
7	14	21	28	35	42	49			
8	16	24	32	40	48	56	64		
9	18	27	36	45	54	63	72	81	
10	20	30	40	50	60	70	80	90	100

MEMORIZATION OF MULTIPLICATION . . .
EXERCISE #4. . .

Presentation

1. Introduce Chart IV.

1. On Chart IV, the pink numbers on the left are now both the multiplier and the multiplicand.

2. Child takes loose combination and writes it in his notebook.

2. 7 X 9 =

3. Show the child how to find the product on the Chart --- note similarity of the process to the addition Chart IV.

3. To find the product on this chart, first I find the multiplicand 7 with my finger and move it to the very end of the road. Then I place a finger on the pink 9 multiplier and move it along until it is directly below my first finger. . . Which descends to meet on this white square --- 63.

Child writes product in notebook.

And that is the product.

4. Show that if the multiplicand is larger, we must reverse the combination before finding the product.

4. 9 X 8 =
Here the multiplicand is the larger number.
And when we use this chart, we must always begin with our multiplicand as the smaller number.
So we must use the combination 8 X 9 to find our product.
Will our answer be the same for 8 X 9 and 9 X 8?
Then we can proceed with 8 X 9 on our Chart IV to find the product 9 X 8.
Can you find that product?

EXERCISE #5: Bingo

PART A
Material

1. Chart V. Chart V is Chart III without the products written in the white squares.
2. Box of loose combinations.
3. Box of green wooden stamps, representing each product.
4. CONTROL - Charts I & II.

Presentation

1. Put Chart V on the mat. Have the child put out all the wooden stamps with the products face up.

2. Child draws a combination from the box and writes it in his notebook.

3. Then he finds the stamp for the correct product of the combination.

4. He places the stamp on the square of Chart V that corresponds to his combination.

5. IF he does not know the product for the combination, he consults Chart I. If he does not know the correct position on Chart V, child consults Chart III.

6. CONTROL - Chart I for the product, Chart III for the position.

Part B
Material
(the same, without the loose combinations)

Presentation

1. Child puts all the stamps back in the box.

2. Child draws one product from the box --- 30.

3. He writes in his notebook: 30 =
and then he DECIDES which combination to use.
30 = 3 X 10.

4. He places the stamp on the corresponding square for that combination.

5. CONTROL --- Charts I & III.

Part C
Materials
(the same)

Presentation

1. The child finds among the stamps---NOW MAY BE LAID OUT AGAIN--- all the products of one number. Es: He finds the four 20s.

2. In his notebook the child writes all the combinations which give 20: 4 X 5 = 20 , 5 X 4 = 20 2 X 10 = 20 10 X 2 = 20.

3. Then he places the stamps (four 20s) correctly on Chart V, looking for each combination he has written.

4. Continues with the exercise, taking all of one equal product each time until the board is completely filled. Included will be 81 of the white squares---and 19 additional products which cover each of the blue and pink squares. 100 products total.

NOTE: The Bingo Game is an indication of whether or not the child has succeeded in memorizing all of the multiplication combinations.

SPECIAL CASES

As in addition and subtraction, there are six special cases of multiplication:

1) To calculate the multiplier: 2 X ? = 12
"How many times do I have to take 2 in order to get 12?

2) To calculate the multiplicand: ? X 7 = 28.
"What number taken 7 times gives 28?

3) To calculate the product, given first: ? = 4 X 7.
"What is the product is we take 4 seven times?

4) Knowing the product and the multiplicand, calculate multiplier:
28 = 4 X ?
"28 is obtained if we take 4 how many times?"

5) Know product and multiplier, calculate multiplicand: 12 = ? X 3
"12 is the product of which number taken 3 times?

6) Calculate the product with two unknowns: 6 = ? X ?
"24 is the product of which number taken how many times?

EXERCISE A

Material

1. Prepared cards of the special multiplication combinations. A sampling of the six cases in various number combinations, from each of the tables.

Exercise:

1. The teacher places the cards with special case combinations in a basket. The children work with these special cases, using their notebooks to copy the combinations and writing the unknown in red.

2. Child draws, reads: Which number taken 9 times gives 45? Then writes: 5 X 9 = 45

NOTE: It is important during this exercise that the child learn to read the special case combinations correctly as the expression of them is a key to understanding the verbal problems.

3. Teacher mixes with these special cases those of addition and subtraction and the children continue the exercise.

EXERCISE B: Verbal Problems

Materials

Cards on which are written verbal problems expressing the six special cases of multiplication.

MEMORIZATION OF MULTIPLICATION. . .

Exercise

1. Child selects a card: How many eggs must I take seven times to get 28 of them?

2. If the child has understood the special cases, he should be able to formulate the equation: ? X 7 = 28
He writes that in his notebook.

3. Then he writes the equation with the unknown in red: 4 X 7 = 28.

4. A second example: In his pencil case Leo has 15 pens. Rose has given him 3 pens how many times to make this number?
15 = 3 X ? 15 = 3 X 5

5. Mix the verbal problems of addition, subtraction and multiplication. NOW THE CHILD MUST FIGURE OUT THE OPERATION.

NOTE: In the preparation of these cards, the problems should be coded on the back with dots to indicate which operation is needed.

NOTE: The child who memorizes the multiplication combinations in an abstract way has no point of reference when he forgets one. The child who learns them through the use of the materials may recall the correct product in a rational way, with the beads or on the board or with the charts. . .BECAUSE HE HIMSELF HAS CONSTRUCTED THE PRODUCT.

GAMES FOR THE MEMORIZATION OF MULTIPLICATION

GAME #1: Skip Counting

Material

1. The Cabinet of Powers

   The cabinet of powers is a fascinating piece of equipment. In its visual presentation of the powers of numbers, there is found for every number from 1 through 10 the following materials in beads:

   a) The bead bar.
   b) A short chain representing the square of the number.
   c) A square. . .as many squares as necessary to form the cube.
   d) A long chain showing the cube of the number.
   e) A cube.

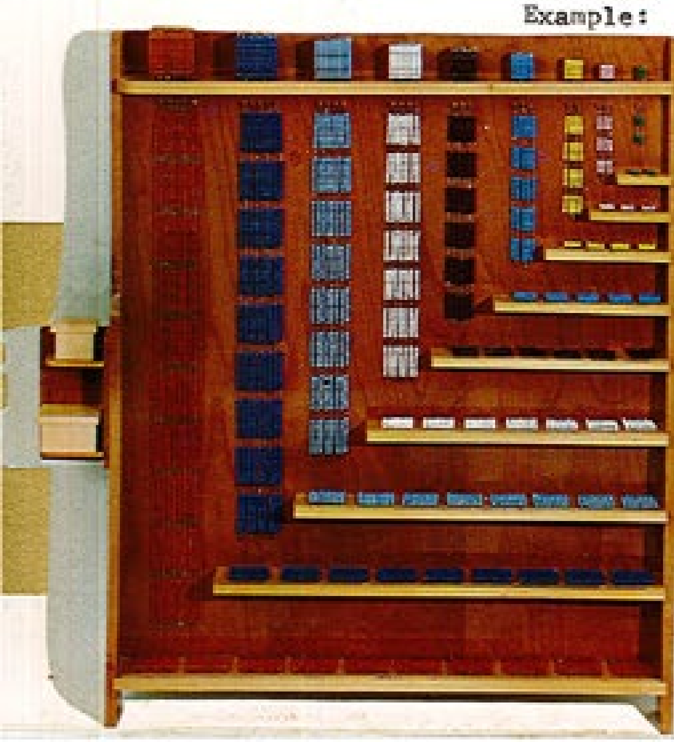
   The colors for each quantity are the same as the colored bead bars: one-red, two-green, three-pink, four-yellow, five-turquoise blue, six-brown, seven-white, eight-sky blue, nine-dark blue, ten-gold.

   The arrangement of the cabinet gives an immediate visual impression of the powers:

   a) The length of the shelves increases proportionally---ending in a position just below the end of the corresponding chain and exactly the length of the correct number of squares.
   b) The chains have rings so placed that when one hangs them, the cube chain forms squares.

2. Two boxes: one containing the arrows for the short chain which include the first numbers of the quantity, then one for each multiple. . .the the square.
   Example: 1,2,3,4,8,12,16.

   one containing the arrows for the long chain, beginning with the quantity numbered from one and then the multiples. . .to the cube.
   Example: 1,2,3,4,5,10,15,20,25,30,35,40,45,50,55,60,65,70,75,80,85,90,95,100,105,110,115,120,125.



MULTIPLICATION MEMORIZATION GAMES. . .
Skip Counting. . .

Presentation

1. Begin by showing the child the red 1 bead.	1. What is this?
2. Then present another number, beginning with the bead bar, and showing all the materials for that quantity, introducing the nomenclature.	2. What is this? (4) This is the bead bar 4. And this is the square of 4. The short chain of 4 is called "the square chain." This is the long chain, called "the cube chain." And this is the cube of 4.
3. Using the short chain, show how it forms a square and display it beside the square.	3. If we fold up the short chain, what figure have we made? Here is the square. Are they the same?
4. Unfold the chain and look at it beside the square.	4. Now the square chain looks like it does in the cabinet.
5. Present the group of arrows for the short chain being considered and show the child how to place the arrows.	5. Here are the arrows to help us count this square chain. Remember how we counted the 100-chain? Here again we begin by placing an arrow for each bead in the first bar: 1, 2, 3, 4. Then we place an arrow at the end of the next bar---8---and 12 and 16. We stop at 16 because that is the square of 4.
6. Move the square to the end of the short chain.	6. Let's place the square at the end of the chain to show that we have the square chain.
7. Ask the child to count the arrows, and then have him count them backwards.	7. Now we can count this chain by 4s. Let's count it backwards, too.
8. Introduce the long chain---much more interesting. First show it beside the short one as a comparison.	8. Now let's look at the long chain. Notice how much longer it is than the short chain.
9. Then set out the cube at the end of the long chain. Show the cube next to the square and then build the cube with the squares.	9. This is the cube which our long chain shows. Look at it beside the square. How is the cube formed? Can we show that?

MEMORIZATION MULTIPLICATION GAMES.
Skip Counting. . .
Presentation. . .

10. Present the arrows for the long chain and have the child place them.

10. With these arrows, we can count the cube chain.

11. Place a square each time he reaches a square and at the end he places a cube.

11. Each time we count a square of the number, we place one of our squares at that point. At the end we place the cube. Then: our square will be placed here: at 16, 32, 48, and at 64 where we will also place the cube.
64 is the cube of 4.

12. The child counts by arrows and then counts backwards.

NOTE: At first the child must count by arrows, but gradually he may be encouraged to learn the progressive numeration (skip counting) forwards and backwards.

EXERCISE #1: Lay out all of the short chains OR
Lay out all of the long chains OR
Make a tower with the cubes.

Indirect Preparation: For work with the powers.

In the exercise, the child observes the comparison. That each set of chains increases proportionally. That the higher the quantity, the larger the increase, the greater the interval between each successive chain or cube.

EXERCISE #2: With the short chains, the child explores the possible geometrical figures he can make: a) chain of 2 forms an angle, b) chain of 3 forms a triange, c) chain of 4 forms a square, d) chain of 5 forms a pentagon.

Indirect Preparation: Gives the child a geometrical vision of the perimeters of regular polygons.

****

Direct Aim: With these exercises, we give the child the names of the square and cube, which he will use for further counting and memorization work. We do not want him to study the squares and cubes.
Also evident in the work here is the linear aspect that is the passage from line to square, square to cube---chains.

Indirect Aim: Preparation for the understanding of the powers of numbers.
Opportunity for the child to absorb the difference between different bars, square and cubes.
Indirect preparation for the child to understand multiples and divisibility.

AGE: 6 on

CONTROL: in number of arrows

GAME #2: The Snake Game of Multiplication

Material (All the same materials used for the addition snake game)

Presentation

1. Explain to the child before he builds his snake that he must use several bars of the same color to form the snake. For Example: 4 eight-bars, 3 six-bars, and 5 seven-bars.

2. The child then constructs the snake of colored bead bars and proceeds to count the snake as he did in the first memorization games for addition, taking two bars down each time to count the snake,

3. When the child has transformed the whole snake, he WRITES the total in his notebook. 85

4. Child proceeds with the check, first placing together the bead bars of the same quantity. . . .
   and we say "There are four bars of 8.
   We can write 8 X 4 = 32. . .

5. Child changes the colored bead quantity for golden bead tens from the transformed snake, changing ten bars when necessary to show the exact quantity for the like group of bars he has multiplied. For 32, he must take three ten bars and a bar of 2.

6. When the child has shown the check, matching the golden bead combinations to the quantity groups, he writes in his notebook the multiplication combinations which show the bead quantity groups and total the snake.

8 X 4 = 32
6 X 3 = 18
7 X 5 = 35
85

GAMES FOR MEMORIZATION OF MULTIPLICATION. . .

GAME #3: The formation of all the multiplications with one multiplicand. Only the multiplier changes.

Material

1. The box of colored bead bars, with the 10-bars included.
2. The box of operations symbols.
3. CONTROL: Chart I or Chart II
   NOTE: Insist that the child checks his work.

Presentation

1. Take one quantity and begin forming the table with the first combination.

   1. Let's begin with 5.

2. The child writes: 5 X 1 = and shows it with one 5-bar. Then he forms the product below vertically. He writes the answer.

   2. 

3. Continue with 5 X 2 =

   3. 

4. The child continues the table, forming each combination through 10 with the same multiplicand and writing the products in his notebook.

5. IF the child does not know the product for the combination, he consults Chart I or Chart II---he DOES NOT COUNT THE BEADS.

6. In this way, the child does each of the tables.

7. When he has done the exercise several times, invite him to cover the work and write the product from memory. Gradually he is able to write both combination and product.

Direct Aim: To bring to the child's consciousness the meaning of multiplicand and multiplier. We see the multiplier only in the number of bars laid out.

Indirect Aims: This succession of multiplications with the materials prepares the child for the concept of surface: one bar is a line, then we have two bars: a surface.

Prepares the child for geometrical figures: line, different rectangles, square. . . .and the fact that we only get the square when the multiplier and the multiplicand are the same.

GAMES FOR MEMORIZATION OF MULTIPLICATION. . .

GAME #4: First multiplication X 10

Direct Aim: To discover what happens to a number when it is multiplied by 10.
To bring to the child's consciousness what it means to multiply a number by 10. . .that is, simply to add a zero to the number.

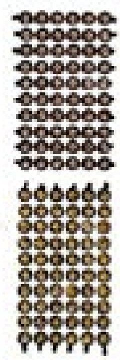
Presentation

1. Ask the child to take ten of any quantity---they all must be the same. Have him form a vertical line with them.

2. Child writes the combination in his notebook and then forms the product vertically beneath. IF he does not know the answer, he consults the table and then forms the product.

3. The child continues with the other quantities, taking ten of each and forming the product, and writing it.

4. If the child does not realize, through the exercise, that the table of ten is formed simply by adding a zero to the multiplicand, help him by observing the combinations and products he has written.

1. 

Here we have 6 taken ten times. That is 6 X 10.

What is our product?

Six 10-bars: 60.

GAME #5: How Many Ways We Can Form One Product

Indirect Aim: A preparation for factoring.
A preparation for the study of multiples, and divisibility.
A preparation for the memorization of division.
A preparation for the study of prime numbers.

Presentation

1. Write a product on a slip: 12.

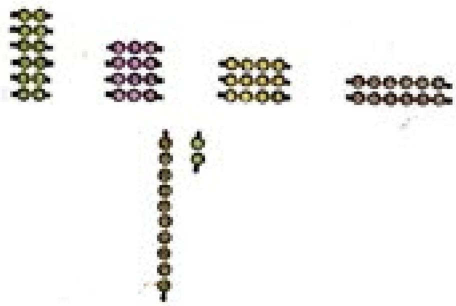
2. Note that we know we can form 12 with twelve 1s; begin with 2-bars.

1. Let's see how many ways we can form 12.

2. First we will form the 12. We know that we can make the 12 with twelve 1s. . . So let's begin with 2. Can we form 12 with 2-bars?

GAMES FOR MEMORIZATION OF MULTIPLICATION. . .
GAME #4. . .Factoring. . .

3.



3. I have been able to form 12 with 2-bars.
How many 2-bars?
Then 2 X 6 = 12.
Can I form 12 with 3-bars.
How many 3-bars?
Then 3 X 4 = 12.
I have also been able to form 12 with 4-bars: 4 X 3 = 12.
And with 6-bars: 6 X 2 = 12.

4. Each time a factor is discovered, the child writes the corresponding combination in his notebook.

5. Proceed to another number. And continue the exercise until each quantity has been shown as its factors.

6. In each case, all the possibilities should be tried: above, the 5-bars are tried giving first 5, then 10, then 15---at which point we discover that the total we are seeking has been exceeded and therefore will not give us 12.

7. The child discovers that some numbers can be formed in many ways, some very few, and some by NONE.

8. CONTROL: After each quantity has been shown as factors, the child consults Chart I and looks for all the combinations that give the quantity he has worked with.
If he chooses 13, he realizes there are no two factors which give 13---AND he cannot find that product on the Chart I.

GAME #6: Small multiplication: the multiplicand, the multiplier and the product in one visual representation

Direct Aim: To further abstract the multiplication, now without showing the product in the bead representation.

Presentation

1. The child takes several bars of one quantity---not more than 10.

2. He places one bar down on the mat:  and reads: eight taken one time is eight. He writes: 8 X 1 = 8.

3. He adds one bar at a time then, reads the multiplication from the number of bars displayed and writes the combination and product in his notebook.

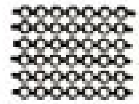
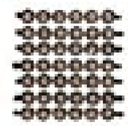
4. CONTROL: Chart I

GAMES FOR MEMORIZATION OF MULTIPLICATION. . .

GAME #7: Inverse products

Direct Aim: To emphasize the value of two factors of a multiplication.

Presentation

1. Ask the child to take several bars of the same kind and form a multiplication.	1. 
2. The child counts the number of bars and writes the combination and product.	2. How many bars do you have? Then you have taken 7 six times. 7 X 6 = 42.
3. Ask the child to take the inverse combination.	3. Suppose we take 6 X 7 instead of 7 X 6. 
4. Note that the results are the same, but the figures are formed in different ways.	4. What is the product of 6 X 7? Then the results of both combinations are the same. What figure does each make? How are they different? Why?

EXERCISE #8: <u>The Square of Numbers from 1 - 10.</u>

Material: the same as for those exercises preceeding, PLUS the squares of the numbers.

Presentation

1. Have the child form all of the multiplications with identical multiplicand and multiplier. Child writes each multiplication in his notebook.	1. Today we are going to take the combinations that are formed with the same multiplicand and multiplier. How will we form 1 X 1? Then write in your notebook: 1 X 1 = 1 How will we form 2 X 2?  2 X 2 = 4
2. Child forms all the squares in a line on the mat. Observe the figures formed.	2. Let's see what figure has been formed.
3. Impose the squares on top of each group.	3. We can check to see if we have really formed squares with these squares. Yes, it is right.

4. Children enjoy coloring the squares on squared paper. . .to keep and show.

THE POWERS OF NUMBERS

Direct Aim: To help the child understand the laws which govern the powers of numbers and to make him aware of the first three powers---10^1 10^2 10^3. . .of the first ten numbers of the natural series of numeration.

Material

1. The cabinet of powers.
   There must be a bar on every shelp for the quantity.
   At this point we begin to call the chains: the SQUARE CHAIN and the CUBE CHAIN.
2. Two boxes: each containing 10 envelopes.
   a. One box for the square chain, each envelope containing 3 arrows in the color of the bead quantities.
      1)base number (2). A smaller arrow.
      2)the square of base (4). Reverse side square combination (2 X 2).
      3)the square of base (4). Reverse side square (2^2).

   b. One box for the cube chain, each envelope containing 4 arrows in the bead quantity colors.
      1)base number (2). A smaller arrow.
      2)the cube of the base (8). Reverse side cube combination (2 X 2 X 2).
      3)the cube of the base (8). Reverse side cube combination using the square (2^2 X 2).
      4)the cube of the base (8). Reverse side cube (2^3).

NOTE: With this material we want to give the child the possibility to intuit the ways in which the numbers can be grouped. . . and to understand the different hierarchies of numbers.

This way of grouping numbers in hierarchies is a conventional system established by man. We must help him understand that these are artificial groups arranged by man. The great rule governing numeration: the obedience to the law of the group.

Each group is formed by single unities. 3 is formed by 3 units. Therefore, each number represents 1 set, one group. Group 2 is formed by two single units. Group 4 by four single units.

AND higher mathematics calls 1 a set. . .that is, the group of numerosity is 1. It is formed by the number 1.

The law, then, that must first be obeyed is the law of the group. That is, 4 = 1 + 1 + 1 + 1. And we must always come back to this base composed of unities, the set of 4.

In this organization of numbers, the number raised to the first power; that is, 10^1, represents the lowest social class, that one composed of the simple citizens, the units. The second power, 10^2 we call the princes, and the third, 10^3, the kings.

THE POWERS OF NUMBERS. . .

First Power 10^1

The first power is 1. It is formed of 9 common citizens and includes the 1 with the zero.

$$10 = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 10^1$$

This is not only the lowest class of numbers, but the lowest class of society. However, it gives the power to the next hierarchy.

As we move up the hierarchy, the power increases. . .not because the hierarchy is more important, but because the number below it increases.

That is, the citizenry increases and gives the power to the hierarchy above it.

Why is this 1 so important? What gives importance to the 1 is the zero. With the addition of each zero, 1 increases its power.

With one zero, we have the first power.

$$10^1 = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + (1 + 0) = 10$$

By MATHEMATICAL CONVENTION, it is agreed that $10^0 = 1$

and that $10^1 \div 10^1 = 1$

so $10^2 \div 10^2 = 10^0 = 1$

Second Power

The second power is formed by as many unities as there are in each group. In the group of 4, there are four units

SO in order to obtain the second power, we have to take that group four times.

4 is the first power. The second power will be formed by 4 X 4.

$$4 \times 1 = 4$$
$$4^2 = 4 \times 4 = 16$$

Third Power

Here we must take 4 X the second power.

That is, we must take 4 squares 4 times.

$$4^3 = 4^2 \times 4$$

NOTE: It is important to show that the governing rule is the base. We always multiply by the base, the number of units of which it is composed. The common citizen is important in his own city.

All powers are formed by units put together to form a row or several rows, but always it is the unit with which we build.

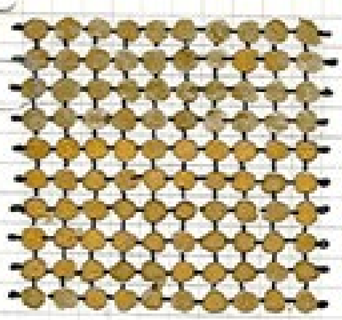
The second power is always a square.

The third power is always a cube. . .and represents, too, the point which begins the next progression of:

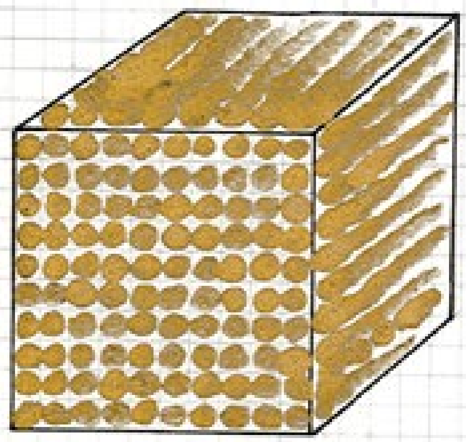
point line square cube.

$10^0 = 1$

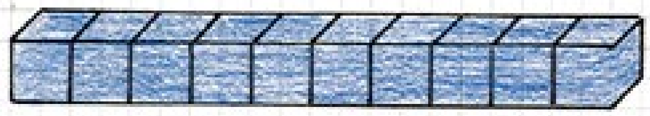
$10^1 = 10$



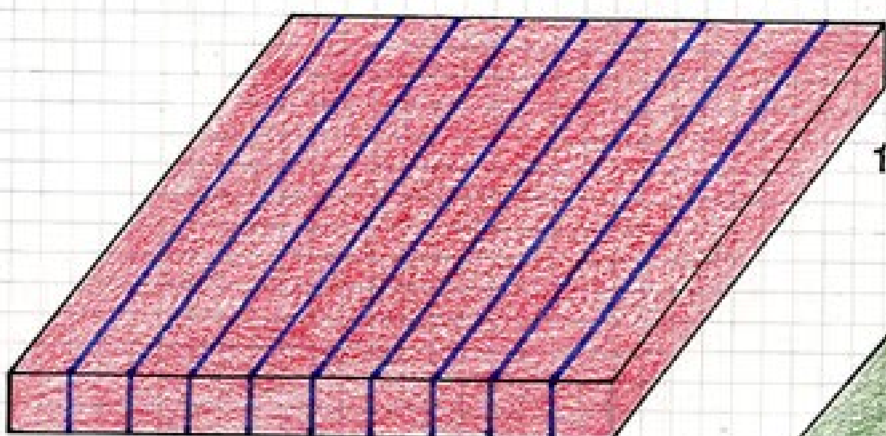
$10^2 = 10 \times 10 = 100$



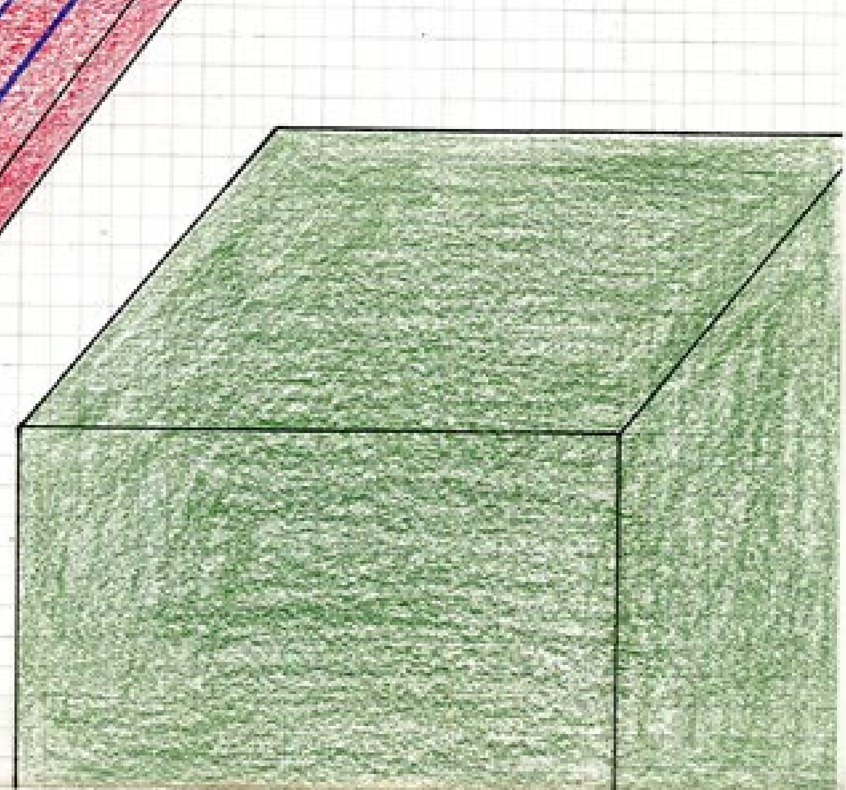
$10^3 = 10^2 \times 10 = 1{,}000$



$10^4 = 10^3 \times 10 = 10{,}000$



$10^5 = 10^4 \times 10 = 100{,}000$



$10^6 = 10^5 \times 10 = 1{,}000{,}000$

THE POWERS OF NUMBERS. . .
Presentation.

1. As in the skip counting exercise, briefly introduce the materials, showing all the materials of one quantity:

1. This is 3.
This is the square of 3. (3 X 3)
This is the short chain.
We call this the square chain.
Let's form a square with the chain.
Is it equal to our square?
This is the long chain.
It is called the cube chain.
This is the cube of 3.
How many squares are in the cube of 3?
Let's form that cube with our squares.
Is this figure the same as the square?

2. Lay out the short chain and show the accompanying envelope with the three arrows. Display them.

2. We have three arrows to help us count this chain.
What is the first one?
Notice that it is smaller than the other two.

3. The child places the first arrow at the end of the first bar, then he places the second one (9/3X3) at the last bead. Finally he places the square at the end of the chain and the last arrow ($9/3^2$).

3. Where shall we place this small arrow?
That tells us what our base quantity is.
Now let's count the square chain by 3s.
We place this arrow 9 at the last bead to show we have reached the square of the 3.
And at the end, let's place the square with this last arrow that tells us 9.

4. Show the reverse side of the first arrow as 3 X 3.

4. How did we obtain this 9?
We counted 3, 6, 9.
There is another way to get 9.
We can multiply 3 X 3.
That's what is tells us on the back of this arrow.

5. Show the reverse side of the last arrow as 3^2 and note the square.

5. We can also write 3^2 because with 3 X 3 we form a square.
It is 3^2 because we are multiplying 3 by a 3 again.

THE POWERS OF NUMBERS. . .
Presentation #2: The Long Chain

1. Introduce the cube chain, forming squares with it and imposing the real squares on top. Put the squares together to form a cube. Then lay out the chain. Introduce the arrows.

1. Here is the cube chain of 3.
How is is formed?
Let's be sure we have squares.
How many squares form the cube?
Now we have four arrows.
Which one is smaller?

2. The child counts the cube chain, as in skip counting: 3 (places first arrow), 6, 9, (places first square), 12, 18, (second square), 21, 24, 27. Places third square.

2. Where shall we place it?

When we reach the square of the number, let's show that with a square.
We need a square at 9.
We need another at 18.
And one at 27, the end of the cube chain.

3. Place the cube at the end of the chain. Then gather the squares to show how the cube is formed and place them in a stack between the end of the chain and the cube.

3. Let's place the cube at the end.
How many squares did we count?
Let's form a cube with them and place them right after the chain.

4. Show placement of the three larger arrows. All are 27.

4. Let's place this arrow that says 27 at the last bead of the chain.
This 27 below the squares.
And this one below the cube.

5. Introduce the calculation for the cube.

Child writes the calculation in his notebook.

5. How did we get 27?
If we got 9 when we multiplied 3 X 3, when we multiply
this 9 X 3
and that = 27.

6. Turn over the first arrow on the reverse side: 3 X 3 X 3.

6. SO 27 = 3 X 3 X 3.

7. Introduce the second arrow reverse which is pointed to the stack of squares and reads: 3^2 X 3.

7. Remember how 3 X 3 can be written?
So 3^2 X 3 is 9 + 9 + 9.
And we can see that we have formed the cube.
3^2 X 3 = 27.

8. Introduce last arrow reverse: 3^3.

8. I can write the cube of 3 in still another way.
We said that 3 X 3 X 3 = 27.
How many times did I multiply the 3?
So I can write 3^3. . . which shows me how many times I must multiply the 3.
I can also call this 3^3 "three to the cube."

Child writes all these calculations.

THE POWERS OF NUMBERS. . .
Conclusion. . .

The child will observe:

1. When we say 10 to the first power, we add one zero to the 1.
2. Ten to the second power is the addition of two zeros.
3. Ten to the third power means adding three zeros.
4. It is the same with 10^6 or 10^{10} ----- we have to add as many zeros as the number indicates.
5. That little number is called the exponent.

It is important that the child gradually come to this understanding. He already knows that, when multiplying a number by 10, he must only add a zero. Here he realizes that he adds just as many zeros as the exponent says.

If he does not begin to discover it,
HELP HIM.

Age: After 8 years.

GAMES FOR MEMORIZATION OF MULTIPLICATION. . .

GAME #9: Multiplication of a Binomial by a Number

Presentation

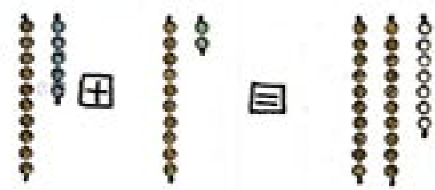
1. Write on a slip: 5 + 4
Then place it on the mat between parentheses.
And form it with bars and signs.

2. Place the multiplication sign and a multiplier after each one. . .on a paper slip.

1-2.

3. Multiply each of the quantities by the multiplier and show the products with beads---as an addition. Proceed with the addition to see the result. ADDING FIRST UNITS AND THEN TENS.

3. What should we do here?
We have to multiply 5 X 3 and 4 X 3.
Then we make:

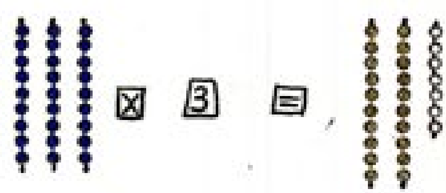


4. Note the number of multiplications made. . .then write on a slip: 9 X 3 = .

4. We have made 2 multiplications.
We obtained 27.
Now let's do this one: instead of 5 + 4, we will use the sum: 9.
9 X 3 =

5. Child forms the new multiplication and compares the product with the first.

5.



6. Later introduce the correct writing, when he has practiced the exercise well:

$$(5 + 4) \times 3 = (5 \times 3) + (4 \times 3) = 15 + 12 = 27 \qquad \text{OR} \qquad 9 \times 3 = 27$$

IT IS THE INTRODUCTION OF THE PARENTHESES THAT MAKES THE DIFFERENCE IN THE FIGURES FORMED WITH THE BEADS.

MEMORIZATION OF MULTIPLICATION

Direct Aim: To give the child reinforcement of the idea that multiplication is a special case of addition (a concept he understood through his work with the decimal system).
To memorize all the multiplication combinations.

Material

1. The multiplication board with 100 bead holes, the numbers 1 - 10 above each row of ten to indicate the multiplier, and a small slot in the left side where the numerals indicating the multiplicand are fitted.
2. A small green box containing: 100 green beads
   small white numeral cards 1 - 10
   a round green counter.
3. The multiplication booklet of printed forms giving the combinations for the multiplication tables from 1 - 10.
4. A green box of loose combinations taken from those forms.
5. A green box containing green wooden stamps --- 100 in number, representing all the products of the tables from 1 - 10.
6. Five Charts

Acquainting the child with the material: Presentation Preparation

Presentation	Preparation
1. Introduce the board.	1. The numbers at the top of each row of holes on the multiplication board indicate the multiplier. What numbers do we have? Here on the left side we have a square slot where we place the numeral card to show our multiplicand. We do our multiplication with these beads.
2. Present a sample multiplication combination.	2. Let's place the numeral card 2 in the slot. It is the number I am going to multiply. I want to take 2 three times. So I place this green counter above the 3. That shows me where I must be when I have finished the multiplication.
3. Place the beads in the small holes---two beads in each of the first three rows for the combination 2 X 3 =	3. Now I begin. I place the counter first above the one and place two beads below it. I have taken two one time. One. . .two. Now I move the counter one numeral to the right, place it above the 2, and put down two more beads. That is two taken two times. Three. . .four. Finally I place the counter above the 3 and put down two more beads. Now I have 2 taken three times. And here is the product. Five. . .six. Two taken three times is six.

GAMES FOR MEMORIZATION OF MULTIPLICATION. . .

GAME #10: Square of the Binomial

Indirect Preparation: For the square of a binomial.
A parallel exercise with the powers of numbers.

Presentation

1. Show the golden square. Tell the child that we are going to divide it into two parts.

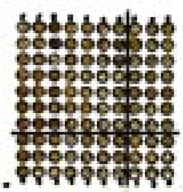
   Child writes 10 X 10 = in his notebook.

   Use rubber bands to divide the square.

   1. What is this?
   This is the square of 100.
   It is 10 X 10.
   We are going to play a game.
   Let's divide this square into 2 parts.
   What parts make 10? (7 & 3)
   Then we will divide this side of 10 first, using a rubber band after 7 to show the two parts.
   And then we must divide the square the other way into 7 and 3.

2. Observe the figures formed.

   Use the squares of 7 & 3 to verify the two squares.



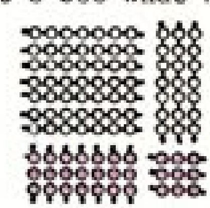
   2. I still have the 100 square, but now it is divided into four geometrical figures.
   What do we have?
   1,2,3,4,5,6,7---we have the square of seven.
   And we have the square of 3.
   Let's check those with the squares of 7 and 3.

3. Note the rectangles and their composition.

   3. We have two rectangles.
   We have 7 X 3 and 3 X 7.

4. Reproduce the square with bars of 7 and 3---showing the four figures. All rectangle bars are placed vertically.

   4. Let's see what has happened:



   We can form our square now with bars of 7 and 3.
   First the square of 7.
   Then the square of 3.
   Then a rectangle of 7 X 3 and a rectangle of 3 X 7.

5. The child now writes the four combinations, moving from the top left.

   5. Let's see if we can write what has happened:

The Square of the Binomial. . .
Presentation. . .

What is this?	7 X 7 = 49
And the rectangle?	7 X 3 = 21
Here?	3 X 7 = 21
And this square?	3 X 3 = 9

IN HIS NOTEBOOK, the child writes: (representing the rows)

7 X 7 = 49	7 X 3 = 21
3 X 7 = 21	3 X 3 = 9

Then :

$$\begin{array}{ccc} 49 & + & 21 \\ 21 & + & 9 \\ \hline 70 & + & 30 \\ = & 100 & \end{array}$$

6. We have formed the square of a binomial.
It can be written in this way:

$$\begin{aligned} 10^2 = (7 + 3)^2 &= 7 \times 7 + 3 \times 3 + 7 \times 3 + 3 \times 7 \\ &= 7^2 + 3^2 + 2(7 \times 3) \\ &= 100 \end{aligned}$$

NOTE: The child should do this exercise a year. . .with all the squares and with all the possible combinations.

GAMES FOR THE MEMORIZATION OF MULTIPLICATION. . .

GAME #11: The Square of a Trinomial

The child is ready for this game only if he can do the following process by himself:

1. Ask the child to prepare the square of the binomial. $(6 + 4)^2$

2. When he has reproduced the square with the corresponding bead bars, he replaces the squares with the real squares.

3. Ask him to write what he has done: 6 + 4 X 6 + 4.
   Show him the use of parentheses: (6 + 4) X (6 + 4).

4. Then the child proceeds with the calculation, according to the figure:

   (6 + 4) X (6 + 4)
   =(6 X 6) + (4 X 6) CRoss out first f
   +(6 X 4) + (4 X 4) Cross out 4.

   = 36 + 24
   \+ 24 + 16
   60 + 40
   = 100

5. The conclusion: 10 X 10 = 100 AND (6 + 4) X (6 + 4) = 100

Presentation

1. Show the child how to divide the 100-square into 3 parts, using two rubber bands on each side.

   1. Today instead of dividing this square into 2 parts, we will divide it into 3.
   Let's use 5 + 2 + 3.
   We must divide first one side and then the other to show
   (5 + 2 + 3) X (5 + 2 + 3).

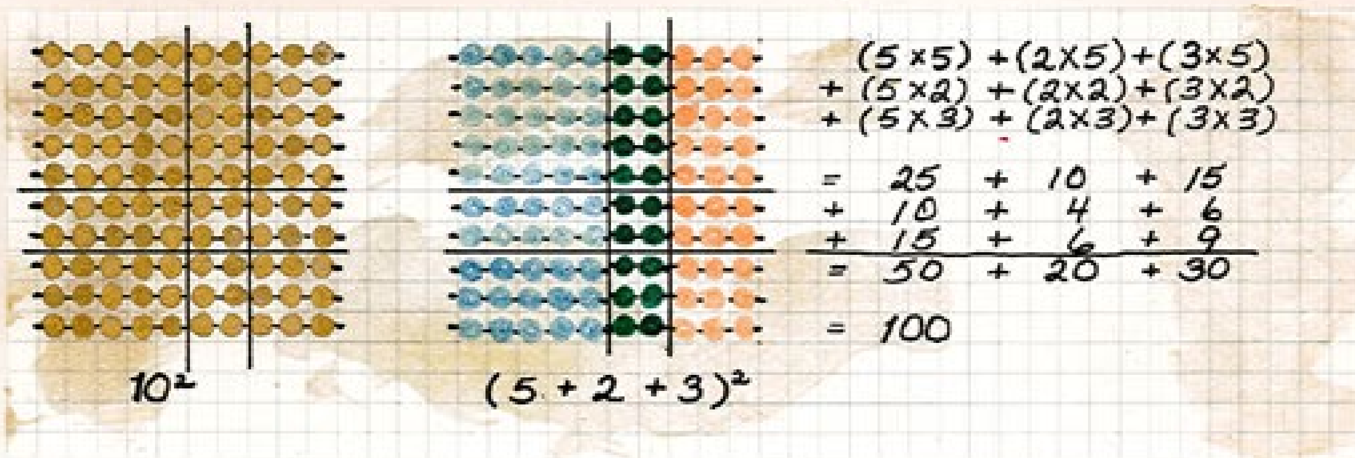
2. Observe the geometrical figures.

   And number of operations.

   2. What geometrical figures can we observe?
   How many squares?
   How many rectangles?
   What is the position of the squares?
   How many operations did we carry out when we divided the square into 2 parts?
   Now I have 9 geometrical figures.
   How many operations will we have?

3. Child forms now the figures of the trinomial with the corresponding bars. As he forms each figure, he writes the calculation.

GAMES FOR MEMORIZATION OF MULTIPLICATION. . .
Square of the Trinomial. . .



4. Verify the number of operations.

4. We said there would be 9 operations. Do we have that many?

5. Child substitutes squares of numbers with real squares.

5. We can place our squares over the squares of the numbers we have here.
How many are there?
How are they formed? (with same multiplier and multiplicand)

6. Child adds all the products.

6. What is the result of our operation?
Then $(5 + 2 + 3) \times (5 + 2 + 3) = 100$

NOTE: Here the child has only transcribed what he has done with the materials. He works with the binomial and trinomial squares in this way a long time.

7. The child can draw the trinomial square figure on square paper with colors.

SECOND Presentation: How the trinomial square is written.
When the powers have been presented. . .and many of the above exercises.

1. Using the same material, we show the child how to write what he has done:

Our figure is 10 X 10.
BUT what figure is this? A square.
The logical way would be to write : $10^{\square}$.
BUT how many tens have I multiplied?
2
So instead of writing that, we write: 10^2.
With this small number we mean that we have to multiply the number by itself.
Whenever we see a small number in this position, it tells us how many times we must multiply the number times itself.

GAMES FOR MEMORIZATION OF MULTIPLICATION. . .
Square of the Trinomial. . .
SECOND Presentation. . .

2. Show the correct calculation for the trinomial:
When I divided the square into 3 parts, what did I really do?
I multiplied (5 + 2 + 3) X (5 + 2 + 3)
SO I can write : $(5 + 2 + 3)^2$

How many operations did I make?

(5 X 5) + (2 X 5) + (3 X 5)
+ (5 X 2) + (2 X 2) + (3 X 2)
+ (5 X 3) + (2 X 3) + (3 X 3)

So first I write:

$5^2 + 2^2 + 3^2$
+ 2(5 X 3) + 2(5 X 2) + 2(3 X 2)

Do we still have 9 operations?

= 25 + 4 + 9
+ 30 + 20 + 12
55 24 21
= 100

3. Note that we have discovered: that the square of the binomial is formed by two terms, four operations.
What is the square of 2? 4
And with the **trinomial** we had three terms, nine operations.
What is the square of 3? 9

THEN, IF I DO A QUADRANOMIAL, THERE WILL BE FOUR TERMS AND HOW MANY OPERATIONS?
QUINTANOMIAL?

AGE: for the presentation of the binomial and trinomial: $7\frac{1}{2} - 8\frac{1}{2}$

GAME #12: Passage from a square to the following square

Presentation

1. Begin with a little tower constructed of the squares---A VISUAL IMPRESSION.

2. Begin with one square, laying it on the mat; and introducing the problem.

2. I have the square of 4. I want to get to the square of 5. Therefore I must form another square.

3. Child writes first the multiplication for the square shown and the one to which we want to progress.

3. 4 X 4 = 16
5 X 5 = 25

4. Point out how many are needed for the progression.

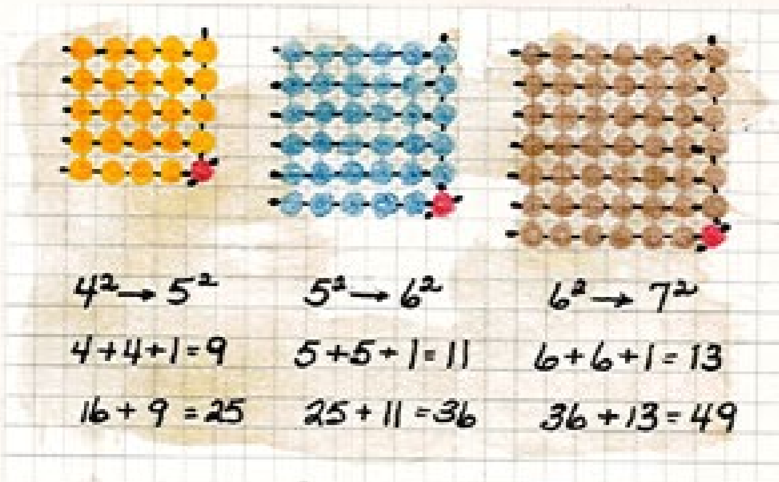
4. If I have 16 and want to form a square of 25, how many will I have to add?
9

GAMES FOR THE MEMORIZATION OF MULTIPLICATION
PASSAGE from One Square to the Following Square. . .
Presentation. . .

5. Try several combinations of bars giving 9. Then show the addition of one bar of the same quantity to each side of the square plus one more.

5. Perhaps I can use this 9-bar.
No. It won't work.
Maybe a 4-bar and a 5-bar.
But then there is not an equal quantity on each side.
And I want to form a square.

Let's add one bar of 4 on one side and one on the other side and a red 1 at the intersection.



Have I added 9 beads?
Do I have the square of 5?
Let's place the square of 5 over it to be sure?
Yes. Exactly.

6. Let's see what we have done: $4^2 \longrightarrow 5^2$ I added two bars of 4 plus 1: $4 + 4 + 1 = 9$.

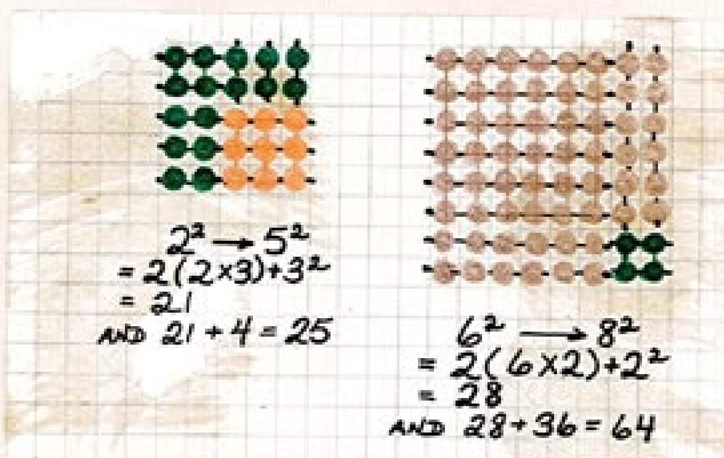
7. Invite the child to find all the squares in succession and write what he has done. He discovers that from one square to the next there is an increase of 2 in the number of beads added for the progression.

GAMES FOR MEMORIZATION OF MULTIPLICATION. . .

GAME #13: Passage from one square to a NON-successive one.

Presentation

1. Present the problem: passage from one square to a non-successive one.	1. Here we have the square of 2. I want to go onto the square of 5. We know that we cannot add bars longer than the side of the quantity bars we already have. So we must add bars of 2.
2. With the bars, build the square.	We add 3 on one side. And then 3 on the other side. I had 2 bars of 2---and I added 3---now I have 5 on one side. And I have 5 on the other side.
3. Child places the square in the space remaining.	But what do I need in this space? I need the square of 3.



4. Explain the process: In order to pass from a square to a non-successive one, we must add to each side of the square as many bars as are equal to the side of that square to which we are progressing. That is, the difference of the sides of both squares. Besides, we must add the square of that difference.

The difference between 2 & 5 is 3. . .so I have to add 3 bars of 2 on each side. And the square of 3.

NOTE: When the child has worked well with this, he is ready to to on to the square of the decanomial.

GAMES FOR MEMORIZATION OF MULTIPLICATION

GAME #14: CONSTRUCTION OF THE DECANOMIAL: In Four Parts

Direct Aim: An aid to the memorization of multiplication.
Indirect Aim: A real preparation for the decanomial square.

Material

1. The bead box of colored bead bars---enough to construct all the tables. 55 of each 1 - 10.
2. The squares for each quantity---enough to construct the cube of each number.

Part A: Vertical Construction

1. Ask the child to first lay out all the bead bars in a row from 1 - 10.

   "What do we have at the top of the multiplication chart I? 1 X 1, 2 X 1, 3 X 1, 4 X 1, ...

   That is what we have shown with the bead bars here across the top of the mat.

   Now we want to construct all the multiplication tables with the beads."

2. The child forms the tables as in figure A. It should be done neatly and carefully, with proper spacing.

3. The construction is vertical; that is, the whole table of the 1 multiplicand is completed first; then the table of 2, etc.

4. When the child has completed the construction, he writes all the tables in his notebook

   OR

   he may prefer to write down each combination or table as he completes it.

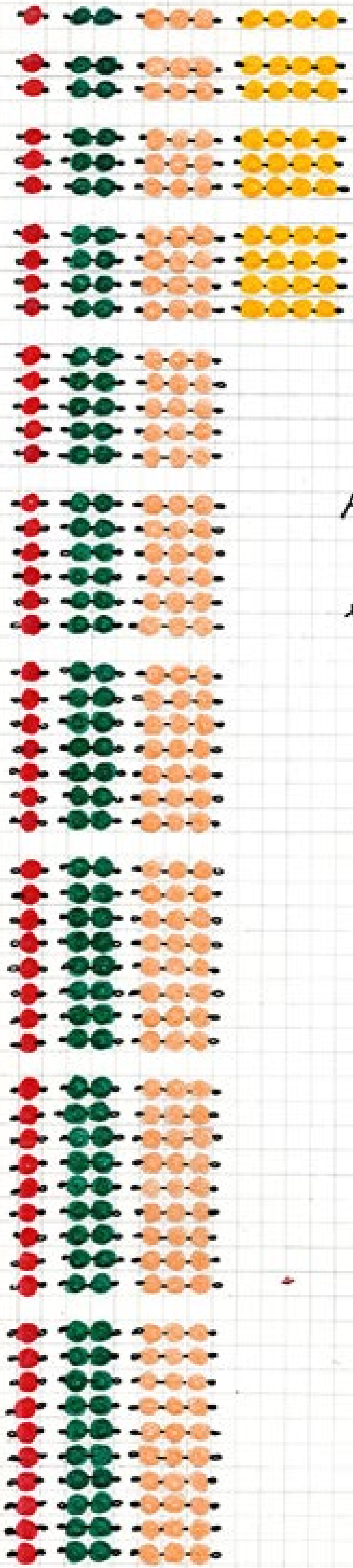
Part B: Horizontal construction

1. Lay out the bars 1 - 10 across the top of the mat as in Part A.

2. The child then constructs the tables horizontally (figure B), that is, he completes the rows across with the same multiplier. First all the numbers multiplied by 1, then by 2, etc.

3. The difference in the construction is also a visual one as the table is built.

RESULT: $(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10)^2 = 55^2$

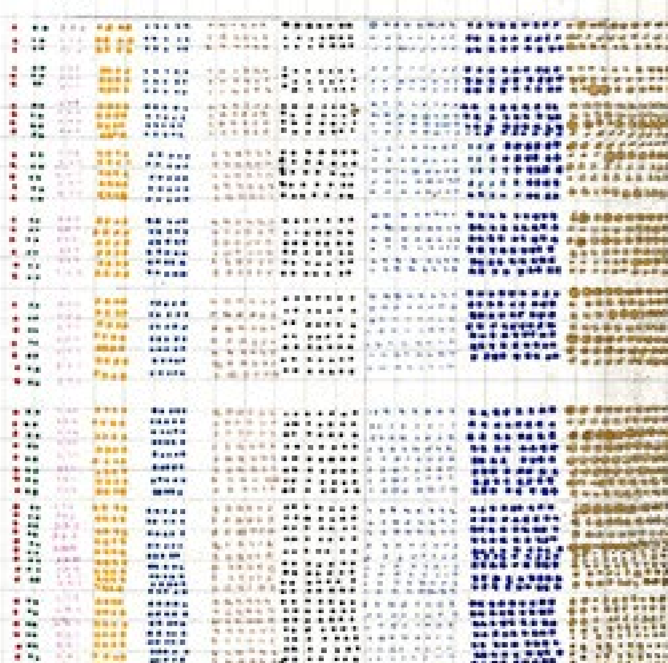


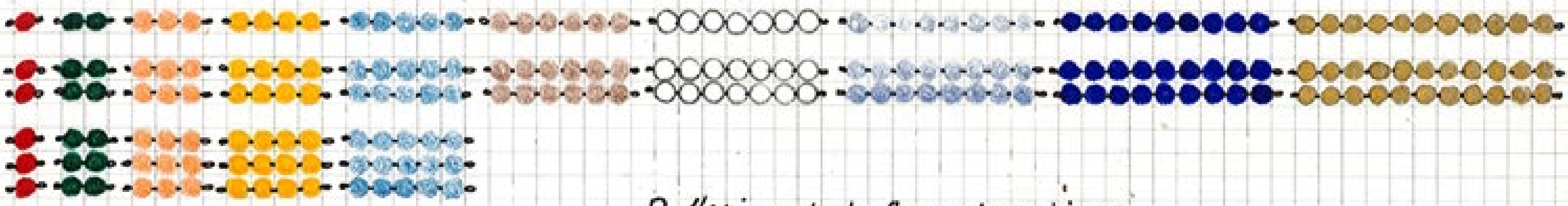
The Pythagorean Tables

A. Vertical Construction
— multiplicand is constant down the line

Sequence:			
1 x 1	then: 2 x 1	and: 3 x 1	and: 4 x 1
1 x 2	2 x 2	3 x 2	4 x 2
1 x 3	2 x 3	3 x 3	4 x 3
1 x 4	2 x 4	3 x 4	4 x 4 ...
1 x 5	2 x 5	3 x 5	
1 x 6	2 x 6	3 x 6	
1 x 7	2 x 7	3 x 7	
1 x 8	2 x 8	3 x 8	
1 x 9	2 x 9	3 x 9	
1 x 10	2 x 10	3 x 10	

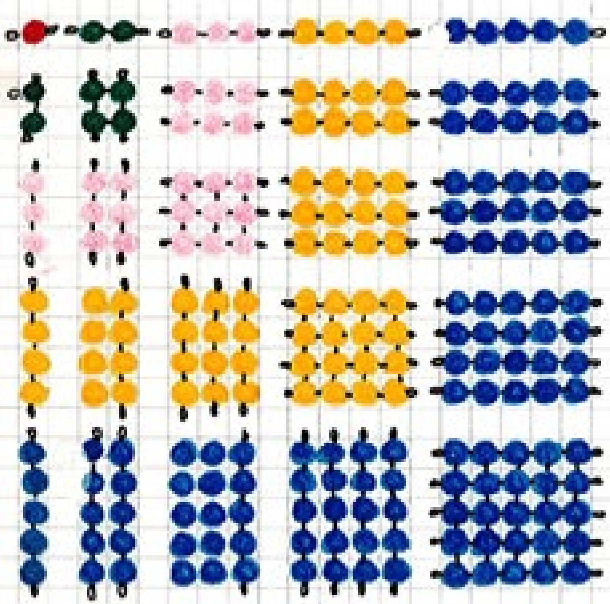
Completed:





B. Horizontal Construction
– multiplier is constant for the whole row.

Sequence: 1 × 1 = 1 2 × 1 = 2 3 × 1 = 3 4 × 1 = 4 5 × 1 = 5 6 × 1 = 6 7 × 1 = 7 8 × 1 = 8 9 × 1 = 9 10 × 1 = 10
then: 1 × 2 = 2 2 × 2 = 4 3 × 2 = 6 4 × 2 = 8 5 × 2 = 10 6 × 2 = 12 7 × 2 = 14 8 × 2 = 16 9 × 2 = 18 10 × 2 = 20
and: 1 × 3 = 3 2 × 3 = 6 3 × 3 = 9 4 × 3 = 12 5 × 3 = 15 . . .



C. Construction from the Angle.

Sequence:
1 × 1
2 × 1
1 × 2 (2 × 1)
2 × 2
3 × 1
1 × 3 (3 × 1)
2 × 3 (3 × 2)
3 × 3
4 × 1
1 × 4 (4 × 1)
4 × 2
2 × 4 (4 × 2)
4 × 3
3 × 4 (4 × 3)
4 × 4
5 × 1
1 × 5 (5 × 1)
5 × 2
2 × 5 (5 × 2)
5 × 3
3 × 5 (5 × 3)
5 × 4
4 × 5 (5 × 4)
5 × 5 . . .

4. Have the child recount the beads to verify the product.

4. How do I know that 2 X 3 = 6? I count the beads.

EXERCISE #1

Material

1. The box with beads, numerals, counter.
2. The multiplication board.
3. The booklet of printed forms.
4. Chart I.

NOTE: The maximum number of products of any numeration system is given by: $(b - 1)^2$. Here that is $(10 - 1)^2 = 81$. This is, in fact, the total number of products which the child must learn.

Montessori goes in with the table of 10 because she wanted to show how easy our system was. That is, the products of the 10 table are identical to those of the table of 1 with a zero added.

1 X 1 = 1		10 X 1 = 10
1 X 2 = 2	AND	10 X 2 = 20
1 X 3 = 3		10 X 3 = 30

SO: The multiplication booklet includes the table of 10. And the children like this last table because it is so simple.

Presentation

1. For interest and clarification (the table of 1 does not clearly give the idea of multiplication) BEGIN with the table of 2 or 3.

1. Let's begin in our multiplication booklet with the table of 3.

2. Establish the multiplicand.

2. What number are we multiplying? Then let's show that multiplicand with the card in the slot.

3. For the first product, place the counter above 1 and then place the beads.

3. What is our first combination? We must take 3 one time. We place the counter above the 1. How many beads will we place in the first row? What is our product?

4. Child writes the product in the space on the form.

4. 3 taken 1 time is 3. You may write that answer now in the booklet.

5. Proceed to 3 X 2 = The beads in the first row are left on the board and another row added. The child counts the total beads for the product.

5. We can leave the first row of beads on the board. What is the second combination? We move the counter to 2. And add another row of 3 beads. What is the product?

GAMES FOR MEMORIZATION OF MULTIPLICATION. . .
Square of the Decanomial. . .

Part C: Construction from the Angle

"Now we are going to form the Pythagorean tables in a completely different way. Each time we are going to form a complete square."

1. Begin, as in Parts A and B, by laying out the bars 1 - 10 across the top of the mat.

2. From 1 X 1, we progress to 2 X 1----
   BUT then we move at an angle to 1 X 2---
   AND instead of formed that with two red 1-beads,
   we show the product 2 X 1 with one green 2-bar.

3. At 2 X 2 we have the perfect square of 2.

4. Proceed to 3 X 1,
   then 1 X 3---but instead of three red 1-beads,
   we form the product 3 X 1 using one 3-bar.
   Then 3 X 2, and on the angle 2 X 3 (3 X 2 and two 3-bars)
   Continue with 3 X 3. . . . and proceed to the fours.

5. The child completes the construction in this way. (figure C)

6. Note the geometrical figures formed.
   "Here we have a point, here we have lines; we have rectangles, and we have squares.

7. The child now substitutes the real squares for each perfect square.

Part D: The Numerical Decanomial : Constructing by the Angle

Material

1. Ten envelopes: Envelope #0 contains 10 blue squares: the first 1 cm.^{2} numbered 1 with the square combination in the left-hand corner: 1^2; the second 2 cm.^{2} numbered 4 and in the corner 2^2--- continuing thru the squares to 10^2, numbered 100, 10 cm.^{2}

   Envelope #1 contains 18 rectangles, 9 horizontal and 9 vertical with a common measure of 1 cm. width. On the rectangles are the products of the multiplication tables.-1-.and so there are duplicates. . .we have the same product two times, but one on a horizontal rectangle and one a vertical.

   Envelope #2 contains 16 rectangles representing du plicate products for the tables of 2.

   Envelope #3 contains 14 rectangles---width 3 cm.---products for the table of 3.

1	2	3	4	5	6	7	8	9
2	2^2 4	6	8	10	12	14	16	18
3	6	3^2 9	12	15	18	21	24	27
4	8	12	4^2 16	20	24	28	32	36
5	10	15	20	5^2 25	30	35	40	45
6	12	18	24	30	6^2 36			
7	14	21	28	35		7^2 49		
8	16	24	32	40			8^2 64	
9	18	27	36	45				9^2 81
10	20	30	40	50				

GAMES FOR MEMORIZATION OF MULTIPLICATION. . .
Square of the Decanomial. . .
Numerical Decanomial. . .
Material. . .

Envelope #4---12 rectangles, products for 4 table.
Envelope #5---10, 5 table.
Envelope #6---8, 6 table.
Envelope #7---6, 7 table.
Envelope #8---4, 8 table.
Envelope #9---2, 9 table.

Exercise:

1. The child begins with the envelope of 1 and constructs the angle, showing 2-10 across the top and 2-10 down the first row.

2. Then he he construct the backbone of the square---the blue squares. These are in envelope #0. He begins by placing the 1^2 - and then continues the diagonal.

3. He continues with the construction from the angle, one envelope at a time.
   NOTE: The child discovers that when he has completed the 5 table, he has almost finished---he lacks only 12 rectangles. . . in the remaining 3 envelopes.

OR

1. The child may take any rectangle, after constructing the blue diagonal of squares, and place it in the correct spot. Or he may take the two like rectangles and place them.

OR

2. He may begin with any square and construct the whole square.

OR

3. He may put all the rectangles in a basket, begin with one and construct the square---but again he needs the diagonal first.

CONTROL: A big wall chart showing the correct placement of the products. This controls the position of the placement.

To find the combinations, the child has only the count the square measures on the graph paper, of which the rectangles are made.

AGE: $6\frac{1}{2}$ - $7\frac{1}{2}$

GAMES FOR MEMORIZATION OF MULTIPLICATION. . .

GAME #14: The Game of Substitution: Moving from the construction of the Pythagorean tables as constructed in Exercise 13: Part C, to a transformation of that decanomial square which is a square composed of squares. . .the number of which, for each quantity forms the cube.

Direct Aim: The sensorial verification that through the construction of the Pythagorean tables, the child has formed the square of the decanomial which equals: the sum of the cubes of the first ten numbers of the natural series.

Point of Consciousness: That the big square constructed with the Pythagorean tables is the decanomial square which is the square of the sum of the first ten numbers of the natural series.

Presentation

1. The child constructs the decanomial square as in Part C of Exercise 13.

2. He first substitutes the squares for the squares laid out in that construction.

3. Then he combines the remaining bar-sets in one of two manners:
   a. Manner #1: moving in two directions from the center square---both up and to the left, he combines the bars needed for the square until he has transformed all bars into squares.
   b. Manner #2: combining those products shown which will give the square and transforming them. . .combining when necessary, products from the two parts of the quantity's angle to achieve the square.

4. Continues through the decanomial until all bars have been replaced by squares. This utilizes all the squares in the cabinet.

5. When the construction is complete, note that, as in the counters, when we pass our finger along the diagonal, there is a square in the way with each odd quantity.

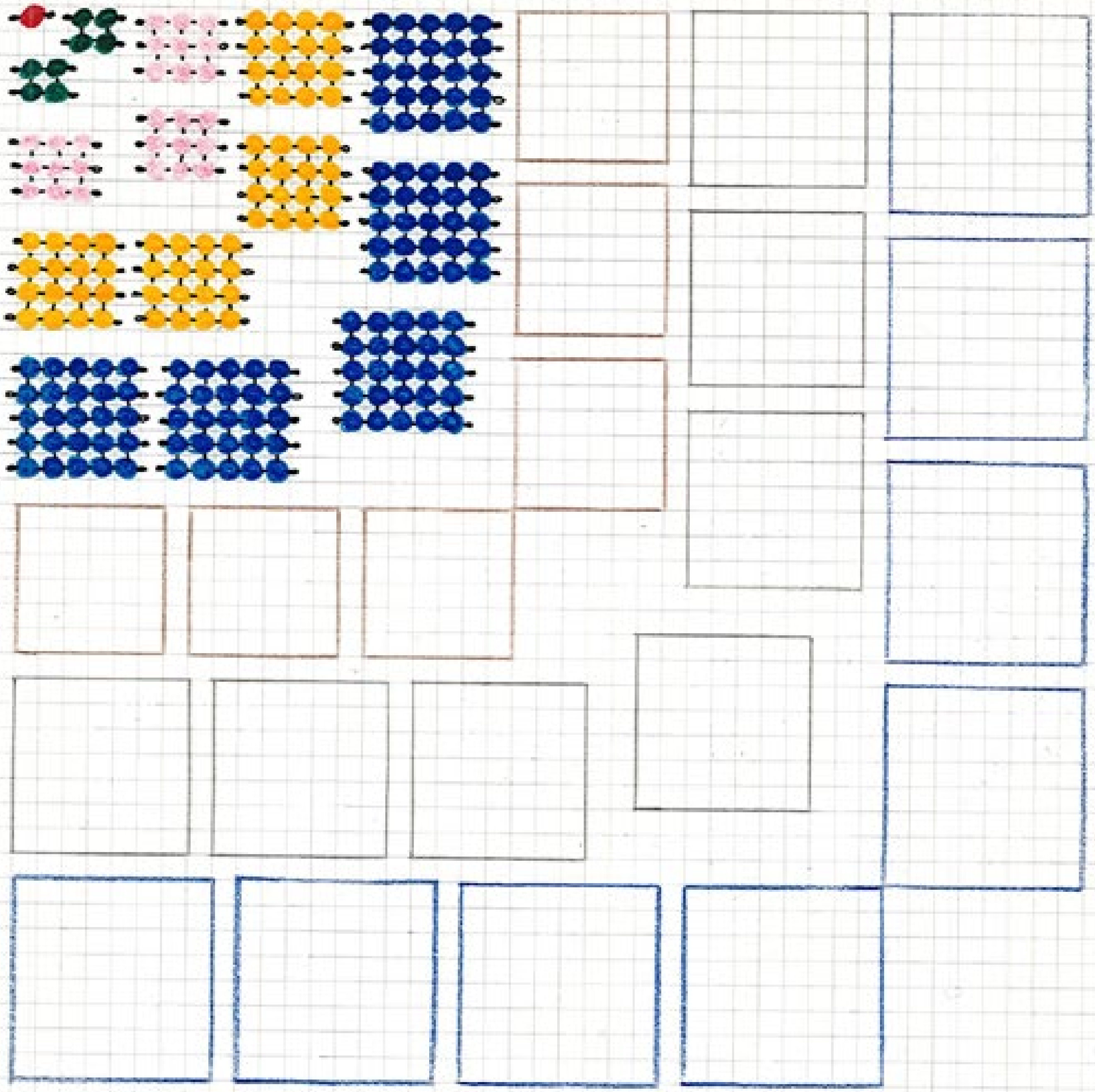
6. Also note that we have transformed the pythagorean tables into a square composed of squares.

7. Finally note that we have in the construction the number of squares which represents the cube of each quantity.

8. The child substitutes the real cubes for each quantity---places them on the diagonal.

9. Then constructs the pink tower with them.

"We have transformed the Pythagorean tables into the pink tower!!"



The Substitution Game

— transforming the decanomial square as constructed in figure C

— the decanomial square becomes a composite of the cube of each quantity represented by the perfect squares.

— the transformation occurring through manner 1 or 2 as noted in the two progressions.

thus:

$$(1+2+3+4+5+6+7+8+9+10)^2$$

$$= 1^3+2^3+3^3+4^3+5^3+6^3+7^3+8^3+9^3+10^3$$

The Substitution Game

Il "decanomio": passaggio dai bastoncini ai quadrati relativi

Formazione dei quadrati di:	1a maniera	2a maniera
1	——	——
2	(2x1) x 2	2x1 + 2x1
3	3x2 + 3x1 3x2 + 3x1	3x1 + 3x2 3x2 + 3x1
4	~~4x3 + 4x1~~ 4x3 + 4x1 (4x1 + 4x1) x 2	~~4x1 + 4x3~~ 4x2 + 4x2 4x3 + 4x1
5	5x4 + 5x1 5x4 + 5x1 5x2 + 5x2 + 5x1 5x2 + 5x2 + 5x1	5x1 + 5x4 5x2 + 5x3 5x3 + 5x2 5x4 + 5x1
6	6x5 + 6x1 6x5 + 6x1 6x3 + 6x3 6x3 + 6x3 (6x2 + 6x1) x 2	6x1 + 6x5 6x2 + 6x4 6x3 + 6x3 6x4 + 6x2 6x5 + 6x1
7	7x6 + 7x1 7x6 + 7x1 7x4 + 7x3 7x4 + 7x3 7x1 + 7x3 + 7x2 + 7x1 7x1 + 7x3 + 7x2 + 7x1	7x1 + 7x6 7x2 + 7x5 7x3 + 7x4 7x4 + 7x3 7x5 + 7x2 7x6 + 7x1
8	8x7 + 8x1 8x7 + 8x1 8x5 + 8x3 8x5 + 8x3 8x2 + 8x4 + 8x2 8x2 + 8x4 + 8x2 (8x1 + 8x2 + 8x1) x 2	8x1 + 8x7 8x2 + 8x6 8x3 + 8x5 8x4 + 8x4 8x5 + 8x3 8x6 + 8x2 8x7 + 8x1
9	9x8 + 9x1 9x8 + 9x1 9x6 + 9x3 9x6 + 9x3 9x3 + 9x5 + 9x1 9x3 + 9x5 + 9x1 9x3 + 9x3 + 9x2 + 9x1 9x3 + 9x3 + 9x2 + 9x1	9x1 + 9x8 9x2 + 9x7 9x3 + 9x6 9x4 + 9x5 9x5 + 9x4 9x6 + 9x3 9x7 + 9x2 9x8 + 9x1
10	10x9 + 10x1 10x9 + 10x1 10x7 + 10x3 10x7 + 10x3 10x4 + 10x6 10x4 + 10x6 10x5 + 10x4 + 10x1 10x5 + 10x4 + 10x1 (10x2 + 10x2 + 10x1) x 2.=	10x1 + 10x9 10x2 + 10x8 10x3 + 10x7 10x4 + 10x6 10x5 + 10x5 10x6 + 10x4 10x7 + 10x3 10x8 + 10x2 10x9 + 10x1.-

This scheme is not for the children

GAMES FOR MEMORIZATION OF MULTIPLICATION. . .
GAME 13: Final Decanomial. . .
Presentation. . .

EXERCISE: The child may start with the pink tower and work backwards towards the tables.

EXERCISE: The child writes the multiplication of every combination of one table and in his notebook writes:

1 X 1 = 1	$1^2 = 1$	$1^3 = 1$
1	1	1
2 X 2 = 4 2 X 2 = 4	$2^2 = 4$ $2^2 = 4$	$2^3 = 8$
8	8	8
3 X 3 = 9 3 X 3 = 9 3 X 3 = 9	$3^2 = 9$ $3^2 = 9$ $3^2 = 9$	$3^3 = 27$
27	27	27

NOTE: At a certain point in this work with the decanomial the child must realize that the big square he has constructed with the Pythagorean square is the decanomial square.

AND

that it is formed by the first 10 numbers of the natural series.

THAT IS

$$(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10)^2$$

He must realize this without help----and at this point, the work is no longer a multiplication memorization, but a new understanding of the decanomial.

NOTE: With the final construction here of the pink tower, the child sensorially veritifes that the square of the decanomial equals:

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3 + 10^3$$

And so we can express the theorem: The square of the decanomial which terms are the first 10 numbers of the natural series is equal to the sum of the cubes of those numbers.

IT IS IMPORTANT

that the child makes the calculation:

$$1 + 4 + 27 + 64 + 125 + 216 + 343 + 512 + 729 + 1000 = 3025$$

AND

$$55^2 = 3025$$

NOTE: The pink tower is a series of 10 cubes from $1cm.^3$ to $10\ cm.^3$. The children, working with the tower, do not learn the square and cubes. . .but it is important that the child works with this scientifically constructed material as a preparation.

Later on the tower is used again for the study of volume; and the child discovers that the numerical value for each bead cube is equal to the corresponding volume of each cube.

That he has 3025 beads.

And that the total volume of the tower cubes is $3025\ cm.^3$ ---cubic centimeters.

MEMORIZATION OF DIVISION

The memorization is a synthesis of the work done with the other operations. Therefore, the child must have all the other work first. Particularly the multiplication.

Through the decimal system work, the child has understood the function of the operation division. He has done division with 1 & 2-digit divisors.

This work done with division will make the child learn all the combinations for division with a maximum dividend of 81 and a maximum divisor of 9.

The division of small quantities by 1-digit divisors constitutes the first exercise in division here. By small divisions, Montessori means divisions where the dividend goes from 1 - 81 and the divisor from 1 - 9. That divisor limit gives us only the simple units 1 - 9 with which to divide.

Material

1. The division board, with 81 holes for beads, the place where the dividend is located. The simple units 1 - 9 representing the divisor are at the top of each of nine rows and colored green to show the units. The quotient numbers 1 - 9 are shown at the left of each column.
2. An orange box containing 81 green beads.
3. An orange box containing 9 small green skittles.
4. The booklet of printed forms containing the division combinations.
5. An orange box of loose combinations.
6. 2 Charts.
7. An orange box containing 81 orange wooden stamps: one for each of the quotients: 9 of each quotient from 1 - 9.
8. Special printed forms. At the top reads "Division" There are four columns, each with 9 spaces. The columns are headed by the words: dividend, divisor, quotient and remainder.

Presentation

1. Introduce the division board, the skittles, beads and the special leaflet.

   NOTE: The maximum dividend in any system is:
   $(b - 1)^2 = 81$

   Maximum divisor is:
   $(b - 1) = 9$

1. On this division board we have 81 holes where we will show our dividend, the number which we will divide into equal parts. The largest dividend, then, that we can have in this game is 81. Across the top is a green strip. What numbers are there? These 9 numbers represent out divisor. In this game we will not use a divisor greater than 9. Why are the numbers shown in green? We will use these green skittles to indicate the divisor---how many parts we will divide the dividend into.

MEMORIZATION OF DIVISION. . .
Presentation. . .

1. . .

On the division board we will read the quotient with the numbers in the left column.
We will use this leaflet to help with out game, too.
It says division. That is the name of the operation.
Then we have four words.
Can you read them.
How many squares are below each of these words?
We have nine squares because we will discover that some dividends can be divided by many divisors and some only a few.
Some have 9 possibilities.
In this box we have 81 beads.

2. ~~Distribute the 81 beads on the board.~~ Show 81 under the word dividend.

2. We'll distribute the 81 beads on our board.
We'll be showing a dividend of 81.
So on our leaflet, under dividend we write 81.

3. Place the 9 skittles on the board. Show 9 as divisor on the leaflet.

Distribute the 81 beads.

3. I want to distribute these 81 beads among 9 skittles.
So I place the 9 skittles at the top to show the divisor.
And below divisor on the leaflet, I write 9.
These 9 skittles are taking the place of 9 children, and among them I will divide the dividend into equal parts.

4. Show the child how to read the quotient and where to write it.

4. Have we divided these 81 bead s equally among the 9 skittles?
Now our box is empty.
How many beads did each skittle receive?
That is our quotient---we can read it here on the numbers at the left.
And below the word quotient on our leaflet we write: 9.

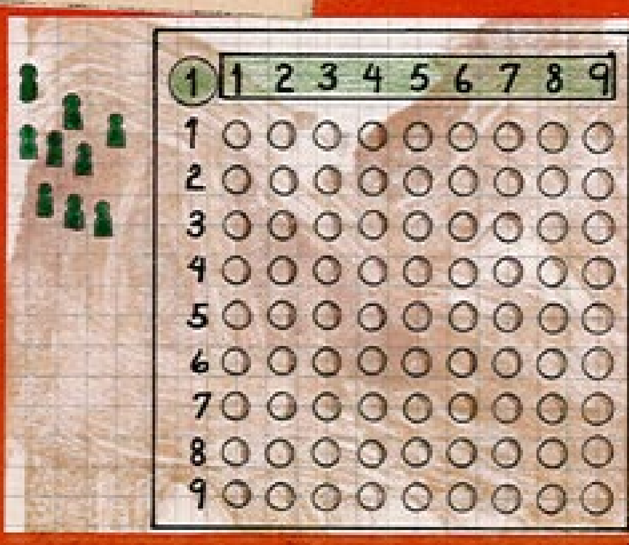
5. Note that this is an important division---have the child underline it.

5. This division is very important---so let's underline it.

NOTE: Montessori notes that the work done by the child to distribute the beads to each skittle gives him real satisfaction---and meets his psychological need for concentration.

DIVISIONE

dividendo	divisore	quoziente	resto
81	9	9	



6. At 3 X 3, the child may note the square figure formed with the beads. If not, at some point during the work, it might be pointed out: Whenever the multiplier and the multiplicand are the same, the figure formed is a square.

7. The child continues through the table, writing each answer in his booklet.

8. Then he checks his work with Chart I.

8. This is the multiplication Chart I. We can use it to check all our multiplication products in the booklet.
Look carefully at the chart.
Where will you look for the table of 3?
What color are the totals---the products?

9. The child continues then to another table. And proceeds with the work until he has completed the entire booklet.

10. The child may like to copy CHART I in his notebook.

EXERCISE #2

Material

1. The board---the beads, counter and numerals.
2. The box of loose combinations.
3. Chart I - CONTROL.

Presentation

1. Lay out all of the numeral cards on the mat so that they can be seen easily, because in this exercise the multiplicand changes.

2. The child draws a loose combination from the box and writes it in his notebook.

3. First he chooses the correct multiplicand from the numeral cards and places it in the slot.

4. Then he places the counter above the multiplier to which he must go for the completion of the combination.

5. He then moves the counter to position above the 1 and begins placing the beads until the combination is completed in beads on the board.

6. Finally he reads the product by counting the beads (or he may have counted them row by row) and writes that product in his notebook.

7. He draws another combination and repeats the work.

MEMORIZATION OF DIVISION. . .
Presentation. . .

6. State the rules of the game.

6. There are two important rules in this game that we must remember.
The quotient cannot be bigger than 9.
The remainder cannot be bigger than, nor equal to, the divisor.

7. Have the child try dividing 81 among 8 skittles.
First he removes the 9th skittle and the 9 beads below it, keeping them in hand.
There is no place to put them.

7. Do you think we can divide 81 among 8 skittles?
Let's take the 9th skittle away and the beads below it.
NO---we see that we cannot divide 81 by 8 in this game because we already have a quotient shown of 9; but we have not distributed all of the beads.
Remember that our maximum quotient is 9.
There is nowhere to put these beads.
And how many do I have?
9 is bigger than our divisor of 8---remember our second rule.
So this division won't work.
We can't do anything with the beads we have left.
And we can't write it down.

8. Conclude that 81 is divisible here only by 9.

8. So we have discovered that we can't divide 81 equally by any other number but 9.

9. Remove one bead---return it to the box to work with the dividend of 80.
Return the 9th skittle.
Leave all the beads placed on the board.

9. Let's put one bead back in the box now and work with the dividend of 80.
Let's begin a new page of the leaflet, too.
And in the first column write 80.
Let's begin by trying to divide 80 by 9.
We will need the 9th skittle again.

10. Put the 8 beads in hand in the column under the 9th skittle.
Then the last row under the first eight must be removed.

10. Let's now replace these beads that we have under the 9th skittle.
Look carefully at the board.
Have we distributed the 80 beads equally to the 9 skittles?
Then we will have to remove the last row.
These beads will be our remainder.
How many are there?
Then we can write 9 under the divisor, 8 as the quotient and 8 as the remainder.

$80 \div 9 = 8$ r. 8

Then I'm not interested in that one.

MEMORIZATION OF DIVISION. . .
Presentation. . .

11. Child lays the remainder out below the board and counts it to determine what it is.

12. Try 80 ÷ 8.

12. Now let's divide 80 by 8.
We must remove the 9th skittle and all the beads under it, then distribute those beads equally among the 8.
What is our remainder?
Then---since our remainder is 8 and equal to the divisor, we can't write that division down.

13. Move to a dividend of 79 by removing another bead to the box. Begin with a divisor of 9, then 8, then 7.

13. Now we will use another dividend, 79.
We must remove one more bead.
And we move to another page on the leaflet.
Let's begin with a divisor of 9.
Can we divide 79 into 9 equal parts?
What is our remainder.
Then we can write that division down, but we're not too interested in it.
Let's try 8.
That is another quotient with a remainder.
Can we divide 79 by 7?
No---the remainder is too big, so we cannot write it down.

14. When the child asks: Why aren't you interested in some. . .

14. We are interested only in those operations in which the dividend can be distributed equally without a remainder.

15. At this point the child has become interested in those important operations. It is a long work. He examines all the possibilities descending through the dividends from 81 - 1. His result is a leaflet of 81 pages that show underlined all those dividends which can be divided equally. He discovers that there are only 36.

NOTE: Because of this discovery, when the child sees the printed division forms, he understands why the pages are arranged in this way, with only 36 pages.

MEMORIZATION OF DIVISION. . .

EXERCISE #1: The Printed Forms

Material

1. The division board, the beads, the skittles.
2. The combination booklet for division.
3. CONTROL - Chart I

Presentation

1. The child may start with any dividend. He selects a page in the booklet for a dividend and shows that dividend as a number of beads in the box. Then, beginning with the first (largest) divisor, he shows that divisor as skittles across the top of the division board.

1. Let's begin with the dividend 7.
We show our dividend with seven beads in the box.
Now, what is our first divisor?
How many skittles do we need on the board?

2. Distribute the dividend beads among the skittles. The child reads the quotient on the board and writes it in the booklet. If there is a remainder, he notes that also on the form.

3. Continue through all those operations on the page.

2. We distribute the dividend among those seven skittles.
What is the quotient?
What does one man receive?
Then we write that quotient in the booklet.
Is there a remainder?
In the second operation, the dividend is the same, but I must change my divisor.
I will have one less skittle.
Now distribute the dividend.
What is the quotient?
And the remainder?

4. The child continues through the work in the booklet.

5. Introduce Chart I---CONTROL for his work.

5. Across the top of this Chart I we find all the dividends which can be divided equally by one or more numbers---81 - 1.
The red numbers at the left are the divisors.
The numbers in the squares (white) are the quotients without remainders.

6. Using the first sequence of divisions done, show the child how to find the quotients & remainders on the chart.

6. In our first problem of division with the dividend 7, the divisor was 7.
We find the dividend on the top row of numbers, then we find the divisor of 7 and bring that finger across the row until it is under the first finger which we now bring down.
The square they meet on has the number 1.
That is our quotient.

MEMORIZATION OF DIVISION. . .
Exercise #1. . .

6. . .using Chart I.

6. In the second division operation with the dividend 7, the divisor is 6.
We do the same thing as before, only this time there is no number in the square where our fingers meet.
So we move to the right until we come to the first number.
Here it is a 1.

7. Note the check of the quotient: the child must know the multiplication combinations well in order to reverse this division process for the check.

7. In order to check our quotient, we can multiply the the quotient times the divisor.
Here that is 6 X 1.
But that gives us 6, not our dividend 7----SO WE HAVE A REMAINDER.
The difference between 6 and 7 is 1, and that is the remainder.
Our quotient is 1 with a remainder of 1.

8. Continue with several examples until the child understands the use of the chart and the multiplication function of the check.

8. 7 now divided by 3.
The first number we meet moving to the right after our fingers intersect is 2. Our quotient.
2 X 3 (the divisor) = 6.
And the difference between 7 and 6 is 1. Our remainder is 1.

9. Note the prime numbers 1,2,3,5,7 in pink on the chart.

9. Look at the numbers which are in pink at the top of the chart.
What are they?
The dividend 1 has as a quotient only 1.
The dividend 2 has a a quotient only 1 and 2.
Which numbers can 2 be divided equally by?
What quotients do you find under 3? 5? 7?
Dividends which can be divided equally only by itself and 1 are called prime numbers.
Later we will discover larger prime numbers.
And we will find that 2 is the only even prime number. All the rest are odd.

MEMORIZATION OF DIVISION. . .

Research For Older Children If They Ask

1. How can we find the 36 dividends?

2. We start with the multiplication Chart IV.
   We do not consider the table of 10 because we will go only to 81, so. . . . .WE CROSS IT OUT. Use squares to show the elimination.

3. In order to go from the products shown to the dividends, we simply eliminate the duplicate products on the chart: 4, 6, 12, 8, 16, 9, 24, 18, 36.

4. The numbers left are the 36 dividends on the Chart I for division and those in the combination booklets.

NOTE: We emphasize going from the product to the dividend because division is always linked to multiplication. Due to this relationship (a reverse one), we can find the dividends from the products.

NOTE: The 36 dividends can also be discovered through a continuous research, starting with the products on multiplication Chart IV. Here we divide the dividends (products) by the multiplier, shown at the left (pink strip) in order to find the quotients.

NOTE: The first method the child used to discover these 36 dividends was the continuous research of all numbers beginning with 81 and down to 1, eliminating those division combinations without even quotients.

Exercise #2: Loose Combinations

Material

1. Chart I---CONTROL
2. Box of Loose combinations, containing only those division operations which have a quotient WITHOUT a remainder.
3. The special division leaflets.

Presentation

1. Begin by explaining what combinations are contained in the box:
   Look at one of the dividend tables:

7 ÷ 7 = 1	
7 ÷ 6 = 1	r.1
7 ÷ 5 = 1	r.2
7 ÷ 4 = 1	r.3
7 ÷ 3 = 2	r.1
7 ÷ 2 = 3	r.1
7 ÷ 1 = 7	

   Only the quotients without a remainder are included in the loose combinations.

2. The child draws one of the combinations.

3. On the leaflet, he writes the dividend (63) and the divisor (7).

4. He finds both on the board, reads the quotient and writes it on the leaflet.

CONTROL: Chart I. . .work with this chart very important as preparation

MEMORIZATION OF DIVISION. . .

Exercise #3: The Bingo Game in Three Parts

Material
1. Chart II: Identical to Chart I, but without the quotients in the white squares.
2. Orange box of wooden stamps, indicating all quotients (9 of each quotient 1 - 9).
3. Box of loose combinations

Presentation: Part A

1. The child lays out all the stamp quotients, face up.

2. Then he draws a combination: 32 ÷ 8.

3. If he knows the answer, he chooses the correct stamp to show his quotient and then places it in position on Chart II.

4. If he does not, he consults Chart I and then proceeds.

Presentation: Part B

1. The stamps are replaced in the box.

2. From the box the child draws one stamp: quotient 7.

3. Then he must decide which combination he will use to show that quotient: 49 ÷ 7. He writes the combination in his notebook.

4. Then places the stamp in position on Chart II to show that combination.

5. CONTROL: Chart I.

Presentation: Part C

1. The child begins by putting all wooden stamps into stamps of like quotients. The result is a square.

2. Beginning with one stack, he writes those combinations which give that quotient, one at a time, placing the quotient stamp in the corresponding square each time. He continues until all those combinations for the quotient have been discovered and the stack is depleted.

3. CONTROL: Chart I.

MEMORIZATION OF DIVISION. . .

PARALLEL ACTIVITIES: Further activities for the memorization of division to be used when the child has worked well with the preceeding activities for a long time.

Material

1. The box of multiplication wooden stamp products.
2. Division Chart I.

Presentation: Looking for the quotients that are even

1. The child begins by arranging the multiplication stamps in stacks, which will be 36 stacks, corresponding to the 36 dividends.

2. The teacher takes one product and, consulting the division Chart I with the child, explores those possibilities for equally dividing that product.

2. Let's discover by which numbers this product can be divided equally. 24
Can 24 be divided equally by 9?
We can find out on Chart I.
No.
Can it be divided equally by 8?
Yes.

3. When divisors are discovered which will give an even quotient, the child writes that combination as a division. And compiles a list:

$$24 \div 8 = 3$$
$$24 \div 6 = 4$$
$$24 \div 4 = 8$$
$$24 \div 3 = 8$$

4. At this point several observations can be made:
The larger the divisor, the smaller the quotient. . .
and vice versa.
The four divisors are also quotients.

5. A check can be made for each combination by multiplying the quotient and the divisor.

Presentation #2: Indirect preparation for finding the highest common multiples and the lowest common denominator.

1. Select a dividend from the 36 on Chart I.

2. Discover with the child the ways in which that dividend may be shown as a combination of prime numbers and multipliers.

$24 = 6 \times 4$ and $6 = 2 \times 3$ $\quad 4 = 2 \times 2$

then $24 = (2 \times 3) \times (2 \times 2)$

$24 = 8 \times 3$ and $8 = 4 \times (2)$ or $8 = (2 \times 2) \times (2)$

then $24 = [(2 \times 2) \times (2)] \times 3$

so $24 = 2 \times 2 \times 2 \times 3$

MEMORIZATION OF DIVISION. . .
Parallel Activities: #2. . .

Direct Aim: The memorization of division linked to the quotients without a remainder and linked to multiplication.

Indirect Aim: Divisibility. It is important to decompose numbers to make later work with the reduction of fractions easier.

SPECIAL CASES

The special cases for division are linked to multiplication. Their particular importance lies in the child's ability to read the special combinations so that he has prepared for those verbal expressions of the same.

1) 48 ÷ ? = 8
2) ? ÷ 7 = 8
3) ? = 35 ÷ 7
4) 5 = 15 ÷ ?
5) 8 = ? ÷ 7
6) 9 = ? ÷ ?

Verbal problems: to be introduced separately first, but soon mixed in with the special case verbal problems of all the other operations.

Example: I give you 4 pieces of candy. In how many parts have I equally divided my 12 pieces of candy?

4 = 12 ÷ ?

THE AIMS

THE DIRECT AIM OF THE BOARD ANDTHE BEADS:
To give the child an understanding of the function of the dividend, the divisor, the quotient and the remainder.
The operation of distributive division reinforced.

THE INDIRECT AIM OF THE WORK WITH THE CHARTS:
To indirectly help the child reali ze among how many divisors a dividend can be divided equally.
And the realization that one dividend may be divided by several divisors.
A preparation for divisibility. How many numbers a quantity can be divided into.

THE DIRECT AIM OF THE DIVISION WORK:
To memorize all the combinations that give the 81 quotients, those necessary 81 operations.
To provide an understanding of how the four operations are linked.

AGE: 6 - 8 years

MEMORIZATION OF MULTIPLICATION. . .

EXERCISE #2. . .

8. Child uses Chart I as CONTROL.

NOTE: It is important in Exercises #1 and #2 that each time the child makes a combination, his product makes a geometrical figure. When his multiplier and multiplicand are the same, the figure is a square. When they are different, the result is a rectangle.

AND: the rectangle of 4 X 6 has a longer base, giving a horizontal rectangle, but 6 X 4 is the rectangle with the same area in a different position. Essentially, they are the same rectangle.

This concept will be emphasized again in the multiplication memorization games.

A Preparation: Moving Towards Chart II (Material - need green slips)

1. Point out the number of combinations on Chart I.

1. It looks like there are alot of combinations to memorize here. How many are there?

2. Recall the elimination work on the Addition Chart I and show how this may be accomplished with multiplication chart.

2. Remember how we were able to eliminate almost half of the combinations on the addition chart? Why could we do that? When we had the same two numbers in our combination, and they gave the same total---then we could cross out one of the two, even though the numbers were in a different order. Why could we not eliminate any of the subtraction combinations? Let's see if it is possible to reduce the number of multiplication combinations.

3. Reduction of the chart may be accomplished with small green slips to cover the inverse duplicates.

3. Let's begin with the first column.
1 X 1 = 1
Is there another combination on our chart like that? No.
Then 1 X 2 = 2.
Can you find another combination using those two numbers that gives 2?
Then we can eliminate the 2 X 1 = 2.
If we learn 1 X 2 = 2. . .we will also know that 2 X 1 = 2.

MEMORIZATION OF MULTIPLICATION . . .
Preparation for Chart II. . .

4. Note the commutative property here, as in addition.

4. This means that if we change the position of the elements in multiplication, the product does not change. . .just as in addition.

5. Proceed with the elimination, using the green slips to cover all the inverse duplicates.

6. Show Chart II, which will be the same visual chart as the I which is now partially covered.

7. Note the squares of the numbers on the top diagonal.

7. There is something especially interesting about Chart II. Each of the combinations on the top diagonal has equal multiplier and multiplicand. The numbers multiplied together are the same.
We call these combinations the squares of the numbers.
In the addition chart, here on this same diagonal we had the doubles of the numbers.

OR

8. The child may copy the whole Chart I, then he can erase the combinations as he is able to eliminate them.

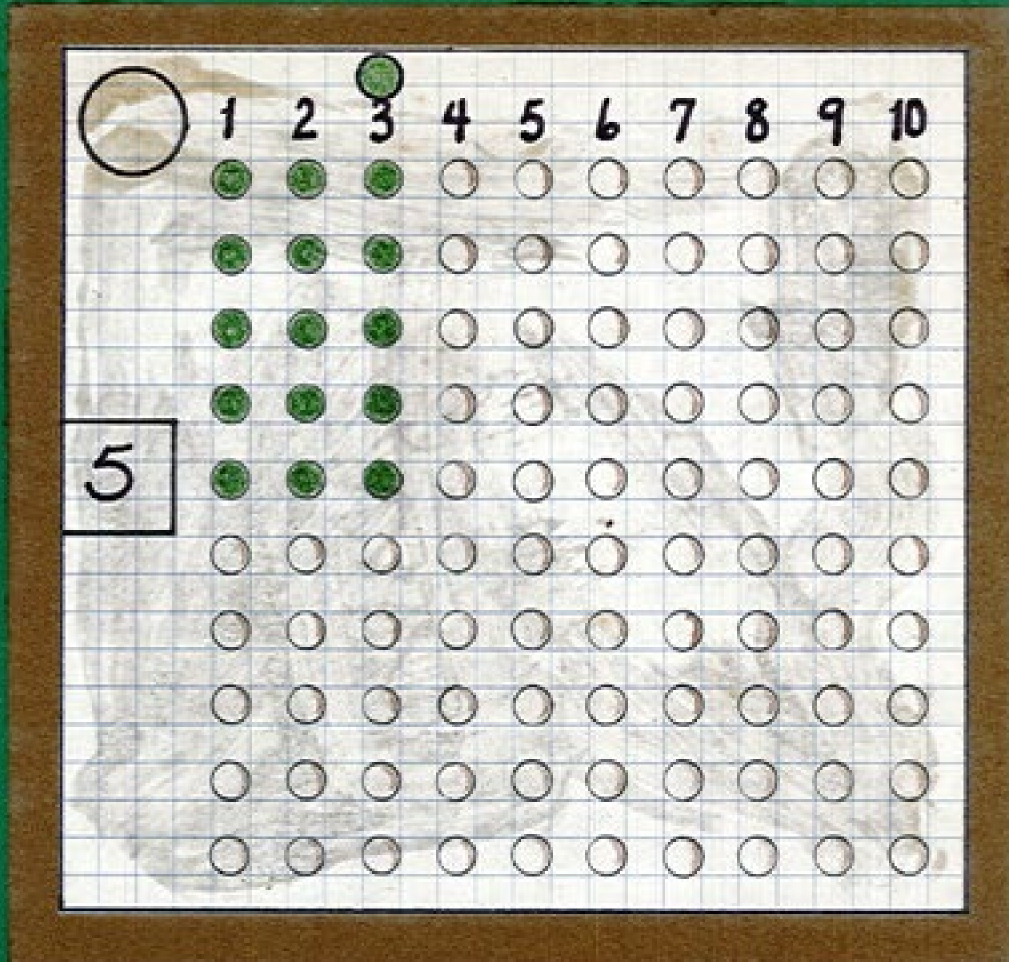
OR

9. He may cross the duplicates out on his notebook copy of the chart.

OR

10. He may cut the chart he has copied leaving only Chart II when he has cut away the duplicates.

5 × 3 = 15



Multiplication									Chart I
1×1=1	2×1=2	3×1=3	4×1=4	5×1=5	6×1=6	7×1=7	8×1=8	9×1=9	10×1=10
1×2=2	2×2=4	3×2=6	4×2=8	5×2=10	6×2=12	7×2=14	8×2=16	9×2=18	10×2=20
1×3=3	2×3=6	3×3=9	4×3=12	5×3=15	6×3=18	7×3=21	8×3=24	9×3=27	10×3=30
1×4=4	2×4=8	3×4=12	4×4=16	5×4=20	6×4=24	7×4=28	8×4=32	9×4=36	10×4=40
1×5=5	2×5=10	3×5=15	4×5=20	5×5=25	6×5=30	7×5=35	8×5=40	9×5=45	10×5=50
1×6=6	2×6=12	3×6=18	4×6=24	5×6=30	6×6=36	7×6=42	8×6=48	9×6=54	10×6=60
1×7=7	2×7=14	3×7=21	4×7=28	5×7=35	6×7=42	7×7=49	8×7=56	9×7=63	10×7=70
1×8=8	2×8=16	3×8=24	4×8=32	5×8=40	6×8=48	7×8=56	8×8=64	9×8=72	10×8=80
1×9=9	2×9=18	3×9=27	4×9=36	5×9=45	6×9=54	7×9=63	8×9=72	9×9=81	10×9=90
1×10=10	2×10=20	3×10=30	4×10=40	5×10=50	6×10=60	7×10=70	8×10=80	9×10=90	10×10=100

All totals should be red.

Shading moves towards Chart II.

MEMORIZATION OF MULTIPLICATION . . .

EXERCISE #3: Chart III

Material
1. Chart III.
2. Box of loose combinations.
3. Paper, child's notebook.
4. CONTROL - Chart I

Presentation

1. Introduce The Chart III. NOTE: on the center diagonal are all the squares of the numbers. . .and this diagonal marks the symmetry---on both sides of it we have the same number. The child will discover this through his work.

1. Just like our multiplication board, on Chart III we find the number we are multiplying on the left side. Here this multiplicand is pink. And the multiplier is again across the top. What color is it? There is only one 1 on our chart. When we multiply 1 X 1, what is the product? Then this 1 represents the multiplicand, the multiplier and the product of that combination. The products of all the combinations can be found in the white squares.

2. Child draws a combination and writes it in his notebook.

2. 6 X 5 =

3. He finds the product on the Chart III, in the square where his fingers intersect.

3. First I place my finger on the multiplicand, the 6 at the left. Then I place another finger on the blue 5, the multiplier. Now I move them together until they meet on a white square. That is the product---30.

4. Child writes the product in his notebook.

5. Checks his work with Chart I.

EXERCISE #4: Chart IV: The Half Table

Material
1. Chart IV: parallel to Chart II, but only the products given.
2. Box of loose combinations.
3. CONTROL - Chart I.