

Presentation

NOTE: Here we are giving the first rule of the decimal system. We go only to the number nine, indicating that there are only nine loose units. With one more, we make a ten. These nine numbers plus the zero form the decimal system.

NOTE: The zero has no beginning and no end, but it marks the beginning and the end. It is the most important element in the decimal system, the base. With it we can make a number infinitely small and infinitely big. We begin to inculcate this idea here.

1. Begin by presenting the two boxes, placing them carefully together so that the numbers follow a neat line. And then, ask a child to read the quantities.
2. Introduce the exercise, saying that we shall put the number of spindles that the number says in each compartment.
3. First point to zero and ask: What shall we put here? Then proceed to the one and ask: What shall we put here? And then to the two.
4. When the child places two spindles in the two compartment, we say: Here we have two separate things. We can make this now into one whole two, into one thing. Let's put them together with a rubber band. Now this is a two. And replace it in the two compartment.
5. Proceed until all compartments are filled correctly, and the spindle container is empty.
NOTE: If there is a mistake, if the jar is not empty or we are short when we arrive at nine, it is a good opportunity to go back and recount each compartment to find the error.
6. Now say: What do you think comes after 9?
How is the ten formed?
Yes. . .so we are going back to the one. . .but it is no longer a one by itself, but a one and a zero.
By adding a zero to the one, we make ten, a new quantity.
7. Reinforce this progression to ten by taking the nine and putting it with the one spindle. Ask the child to count them together to show that it is the next number. Then have him separate them and replace them in their correct position.

Purpose

DIRECT AIM The numeration from 1-10.

THE COUNTERS

Materials

1. The white number cards (cardboard) from 1-10.
2. Small discs in a box numbering 55. (This could be something other than discs such as beans, but must be all of the same kind, regular.)

Presentation

1. Place the cards at random on the table and ask a child to put them in order.

2. Then say: How many discs should we place under this number, (indicating the one).
Ask the children one by one to count out and place the correct number of discs under each number.
 3. The control is an empty box at the end. If it is not empty or there are not enough, ask for a recount, or this might be a good time to go on to the next step yourself which is. . .
 4. The rearrangement of the discs in even rows, placing the odd one in the odd numbers in the middle at the bottom.
Or say: To be sure our counting is right, let's arrange them neatly like this.
And proceed to do so, with their help if possible.
 5. Ask the child to observe the rows: Do you see something peculiar, or different about our rows of discs?
 6. Probably not. Then say: I will help you discover it. Pass your finger between the two rows.
What did you observe?
What happened?
 7. "Tell me which are the numbers where your finger was stopped."
Then have the child count, as you point: 1, 3, 5, 7, 9.
and: 2, 4, 6, 8, 10.
 8. "I will tell you something: those numbers where your finger could not pass thru are called ODD numbers. Those numbers where your finger passed easily all the way down thru the rows are called EVEN numbers."
 9. "Show me an even number."
"Show me an odd number."
"What is this?" (even)
"What is this?" (odd)
 10. Now ask the child not to look, or better, to close his eyes and tell you an even or odd number. Then look to check the discs for control, again passing his finger if necessary.
 11. Together now read first odd, then even numbers in succession.
 12. Ask a child to turn over, face down, the odd numbers and name them. Then the even.
 13. Take away the discs and repeat any one of the preceding exercises for reinforcement.
- EXERCISE: Put the cards aside. Take a random number of discs and arrange them in rows. Ask the child whether it is an odd or even number. . .and why. A big quantity is good.

Purpose

- DIRECT AIM: 1. To verify if the child knows all numerals 1-10.
2. To help the child towards abstraction. (when he puts cards in order without control.)
3. To give knowledge of even and odd numbers.
- INDIRECT AIM: 1. To prepare the child for understanding all even and odd quantities.

A GAME

(an exercise to reinforce the presentations of the counters and spindles)

Materials

1. The white number cards, or another set of similar cardboard cards with the numbers 0-9.
2. Objects all of a same kind in a container.

Presentation

1. Present the cards at random on the table and ask the child to arrange the numbers in order.
2. Present the objects in container and ask the child to place the correct number of objects below each number.

THE SERPENT

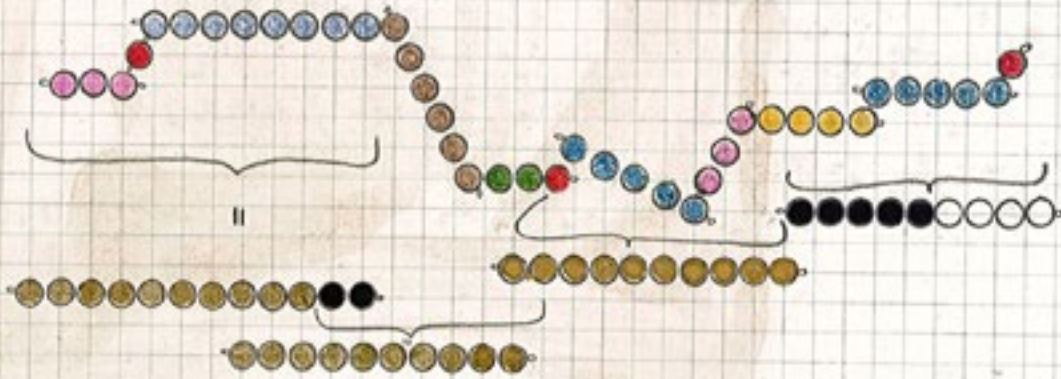
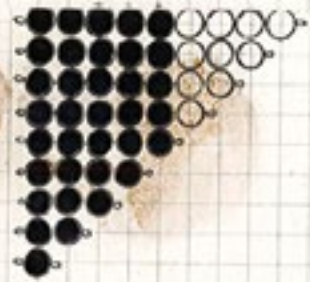
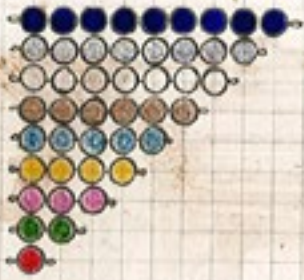
The Search for Ten

Materials

1. A box containing colored bead bars from 1-9, with nine compartments, each containing one of the numbers:
 - 1 - red
 - 2 - green
 - 3 - pink
 - 4 - yellow
 - 5 - turquoise blue (azzurro)
 - 6 - brown
 - 7 - white
 - 8 - sky blue (celeste)
 - 9 - blue, navy blue
2. A smaller box containing the black and white stair, a set of bead bars from 1-9, the first five of which are black and then 6-9 adds white beads, first one, then two and so on.
3. A box containing golden 10-bead bars.
4. A small paper bridge. (a rectangle 5 centimeters in length with a small triangle cut out of the bottom edge in the center. Can be used for indicating the counting as well as marking the end on a ten between beads).

Presentation

1. Begin by presenting carefully the materials to be used. If it is the first time the child has seen the beads, ask him to count each bar as it is presented. Replace all in containers except B&W stair.
2. "Now I will show you how we play this game.
First I will make a snake with some of our colored bead bars."
Then put the colored bead box aside.
3. "Now I will start at his tail and count ten beads".
At the end of the ten, we place the bridge to mark the spot and explain that we will now replace that ten with a golden ten-bar. Having taken the ten-bar out and placed it above the beads we have counted, we say: How many beads are left on this bar?
How shall we show what we had left?
We will place the number we need to show that remainder here, using the correct bar from the black and white stair.



4. Proceed to count the next ten, after having replaced the colored beads in the empty box, (and perhaps shutting the lid to show that those colored beads are gone) As you begin to count the second ten say: I will start counting my next ten beginning right after the last golden bar.
5. Continue the counting and substituting until all colored beads are replaced by golden bars with perhaps a black and white remainder on the end and say: Now my snake has become a golden snake.
6. Then say: Let's find out if the golden snake is exactly as long as the colored snake was.
And open the colored bead box where you have placed all those for which you substituted.
7. First line up the golden bars vertically with space between for comparison.
8. Then have the child help you line up the colored beads vertically in a row beginning with the largest one.
9. Now use the largest colored bar and lay it beside the first ten. Ask the child to count it and then ask him to count, perhaps using the bridge, the ten, stopping with the number of the colored bar. Then he can place the bridge there and count the remainder to see what he needs to complete that ten.
10. If the needed bar is available, show him how to place it next to the golden bar to complete the ten. If not, show him how he can exchange it for the one he needs in the colored bead box, offering plenty of suggestions for counting before and after the change. (It is better to place the largest colored bars away from him so he can perceive more easily the remainder, the completion.)
11. The control of error is an even match between colored beads and golden bars.
12. Finish the presentation by helping the child to replace all beads in their correct positions. The colored beads especially need attention and an error on the teacher's part, brought to the attention of the children for correction is a good help.

Purpose

Reinforcement of the idea of the various combinations for a ten.
Preparation for addition and subtraction.

Age 6

THE DECIMAL SYSTEM

We have already introduced the first cell of the decimal system, that is, the idea of the nine numbers which, together with the zero, will make any other number. Now we turn to the powers of 10. Thus we are giving the child the possibility of forming and counting very large numbers with the nine symbols he knows. The key of the decimal system lies in its simplicity and clearness, and in the ZERO. The key to the system is a matter of organization, of position. We recognize, thru the following presentations, that the absolute value of each number remains the same, but its value within the number we make changes according to its position. Gradually, through our work with the golden beads (units, ten-bars, 100-squares and 1000-cubes), the child learns the value of position and understands the hierarchy of numbers.

KNOWLEDGE OF THE QUANTITIES

Materials

1. Loose golden beads representing units.
2. Golden ten-bars representing, each one, a ten.
3. Golden 100-squares, each square being formed with 100 beads.
4. Golden 1000-cube, formed by 10 100-squares.
5. Supplementary wooden squares and cubes.
6. Some wooden trays, each one with a small square tray that sits within the larger one.

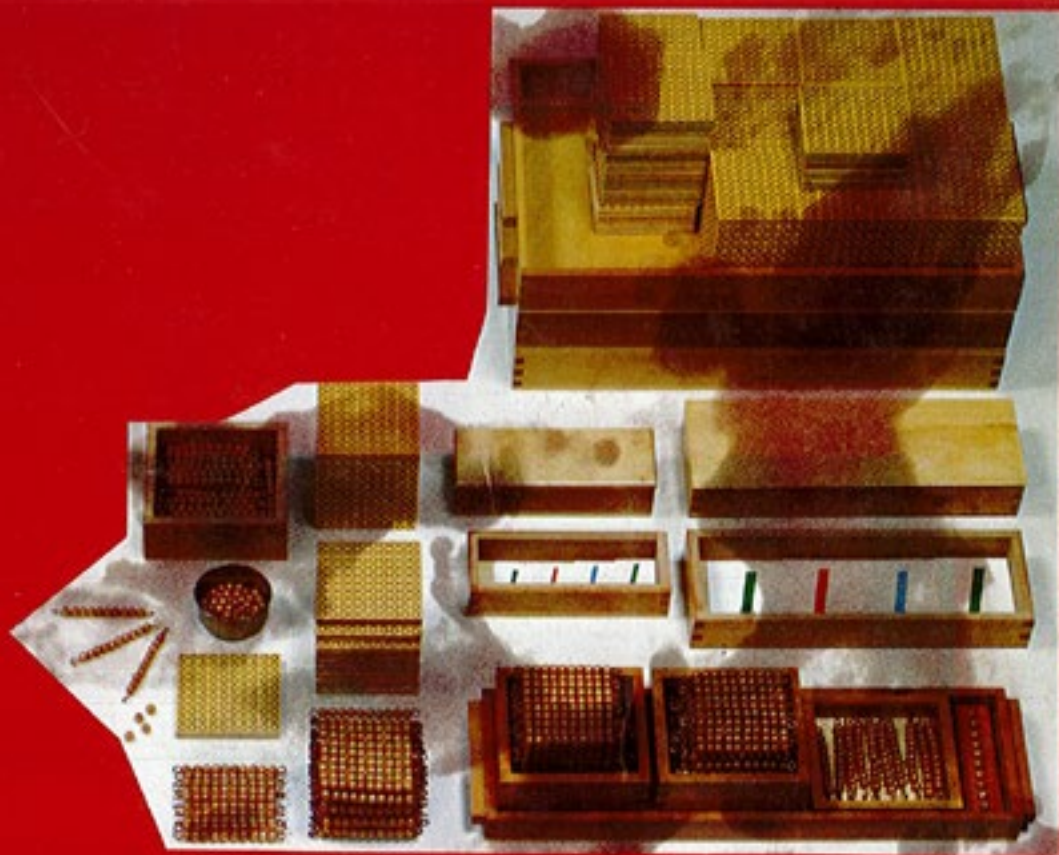
Presentation

1. On a tray, place one loose bead (within the small square tray), one ten-bar, one 100-square and one 1000-cube.
2. Let the child look at them together, and then lay out on the table the two with which he is familiar, saying:
Can you tell me what this is? (indicating the single bead)
Yes, it is one, one unit. We call it the unit bead.
And this, (indicating the ten-bar) can you tell me what this is?
Yes, it is the ten-bar.
Then, taking the unit bead, count with it against the ten bar to reinforce the ten units in it.
Finally, if necessary and helpful to insure the knowledge of the terms unit bead and ten-bar, use the second and third periods: Show me. What is this?
3. Introduce onto the table beside the first two the 100-square and the 1000-cube saying:
Now look, here we have something new.
(Taking the 100-square in hand) You may recognize the shape of this. Take it in your hand and trace it. What is it?
Yes, it is a square and that is part of its name.
We call this the 100-square.
And if you look closely, you will see that it is made of ten-bars. Let's count how many ten-bars there are in our 100-square.

- We have ten 10-bars in our 100-square.
4. "Here is something else new." (pick'ing up the 1000-cube)
 Take this in your hands and feel it. (especially if the child has worked with the geometric solids)
 What shape is this?
 Yes, it is a cube and that is part of its name.
 We call this the 1000-cube.
 Now run the progression saying:
 This is 1, the unit bead, this is the 10-bar, this is the 100-square, and this is the 1000-cube.
 Second and third periods: Show me the 100-square.
 What is this?
 Now have the child observe the 1000-cube closely saying:
 If we look carefully at the 1000-cube, we can see that it is made of 100-squares. Let's count how many squares there are in this 1000-cube. (And using the square as an indicator, count the squares across the top with the child)
 Then, using loose squares, put together 10 to show that the quantity is the same as that of the cube saying:
 Now we can see that 10 100-squares form the 1000-cube.
 5. At this point repeat the information, pointing out:
 The ten-bar is formed with ten units.
 The hundred-square is formed with ten ten-bars.
 The thousand-cube is formed with ten hundred-squares.
 6. Second and third period lessons for reinforcement here:
 Show me the thousand-cube.
 What is this?
 7. Ask the child to close his eyes, and place each of the four pieces in his hand for the tactile experience of the quantity and the sense of its weight. Ask him to name the piece when he has felt it. Then place two pieces, one in each hand, so that he may make the comparison and identify the pieces in relation to each other mentally. A step towards abstraction for him. Finally, as he finishes this exercise, ask him to replace the four pieces on the tray, indicating the position of the unit bead again in the small tray.

EXERCISE:

- a. Bring to the table some additional wooden squares and some wooden cubes and add them to the group of beads, bars, squares and cubes which you have gathered on the table saying:
 Now we are going to make some very large numbers, and so we need some extra hundreds and some extra cubes.
 This wooden square will have the same value as the hundred-bead square.
 And these wooden cubes will each be equal to the thousand-bead cube.
 NOTE: No more than nine cubes should be offered because there is no 10,000.
- b. Ask the child to take a tray and bring a certain quantity of beads to you. Begin with several of the same kind: 5 tens, 6 hundreds, etc. The teacher checks and controls error, asking the child to count what he has brought.



- c. Increase the complexity of the quantity, first including two places, then three, and finally requesting all four. This final exercise, when first given should be given slowly, perhaps one number at a time so that the child can place it on the tray before hearing the next. Gradually his ability to remember the whole request will develop. When he returns or presents his answer including more than one category, the teacher counts it for a check, always naming the higher category first. "And so here we have 2 thousands, 4 hundreds, 6 tens, and 1 unit."

EXERCISE:

(This is essentially the third period of the lesson)

- a. Ask the child to take a tray and bring on it any number he chooses.
- b. "How many have you?"
And perhaps indicating with the hand so that he will tell the highest numbers first, ask him to repeat the quantity he has brought.
- c. If the child brings more than 9 of one kind, show him briefly how he can change his ten for the next higher category, thus simplifying his answer.

Purpose

- DIRECT AIM: 1. To make the child become familiar with different quantities.
2. To give him an idea of the relative size of the different quantities.

INDIRECT AIM: Preparation for geometry---the quantities represent the geometric figures: point, line, square, cube.

Age 6

KNOWLEDGE OF THE SYMBOLS

Materials

1. A box containing the following number cards:
 - the units 1-9, green
 - the tens 10-90, blue, twice the size of the unit cards
 - the hundreds 100-900, red, three times the size of the unit cards
 - the thousands 1000-9000, green, four times the size of the unit cards
2. The wooden square tray.

Presentation

1. Place the four cards 1, 10, 100, and 1000 on the tray and place it on the table. Then take the 1 and the 10, the two numbers the child knows, and place them on the table before him asking:
 - What number is this? (indicating the one)
 - Yes, it is the unit one.
 - And what is this? (indicating the ten)
 - Remember how the ten is made?
 - With a 1 and a zero.
2. Now introduce the two new quantities saying:
 - Here we have something new.
 - This is one hundred.
 - How many zeros does it have?

And this is one thousand.

How many zeros does it have?

3. Second period lesson: Show me ten.
What color is it?
Show me one hundred.
What color is it?
How many zeros does it have?
4. Third period: What is this?
5. Now put the cards at random order and repeat the second and third period lessons.
6. Then repeat the zeros lesson, perhaps turning the card face down the second time thru and asking (with your hand on the card):
What is this?
How many zeros does it have?
7. Finally ask as you arrange your four cards in hand, all flush to the left side:
How many zeros are there in 1? in 10? in 100? in 1000?
8. Slide the cards flush to the right, calling attention to the magic transformation and ask:
Now you must tell me how many zeros are hidden in this number? (indicating 10)
And in this number? etc.
9. Pointing first now to the unit ask:
What number is this? (1) What color is it?
Then indicating the 10:
What number is this? (1)
But here it is called one ten...or we must call this 10.
Remember that it is blue.
What number is this? (indicating 1 of the 100)
But here it is called one hundred.
It is red.
And what number is this?
Here this is called one thousand.
It is green.
10. Now, proceeding from left to right, repeat:
This is one thousand.
This is one hundred.
This is one ten.
And this is one unit.
Ask the child to repeat it in sequence from left to right.
11. Ask the child how many zeros are in each of the categories without looking. And say: The zeros give the value to the one.
12. Ask the child to arrange now all the cards in order: help him begin so that the thousands row is on the left, then the hundreds, tens and units.

EXERCISE:

- a. Ask the child to bring on a tray a certain number, beginning with the simple: Bring me nine tens. When he returns, ask him what he has brought. (second and third period lessons)

The Presentation of the Symbols

1-2.

1

1 0

1 0 0

1 0 0 0

5

1 0

1

1 0 0 0 1 0 0

6.

1 0

1

1 0 0 0 face down

7.

1 0 0 0

8.

1 1 1 1

12.

1 0 0 0

1 0 0

1 0

1

2 0 0 0

2 0 0

2 0

2

3 0 0 0

3 0 0

3 0

3

4 0 0 0

4 0 0

4 0

4

5 0 0 0

5 0 0

5 0

5

6 0 0 0

6 0 0

6 0

6

7 0 0 0

7 0 0

7 0

7

8 0 0 0

8 0 0

8 0

8

9 0 0 0

9 0 0

9 0

9

- b. Increase the complexity of the request, first with two categories, then three, and finally four. When he returns with these numbers, check for him the first few times, naming the higher quantities first so that he develops the pattern of reading first the thousands, then the hundreds, the tens and the units.
- c. When the child has progressed to the exercise in which he brings all four categories, show him how to arrange these cards, beginning with the thousands and all flush left. Then ask:
 - Is this the number I asked for?
 - Then slide them again, as before in the presentation, to the right flush position and ask:
 - Is this it?
 - And in subsequent checks let the child play this magic game of transformation. This is always a good opportunity to ask again:
 - How many zeros are hidden here?
 - And here?
 - Or:
 - Where are the units?
 - What color are they?
 - Or:
 - What is this?

Purpose

DIRECT AIM: To make the child sure of all the symbols, from 1-9000.

Age 6

MATCHING THE SYMBOLS TO THE QUANTITIES

In this exercise we are presenting the general overview of the decimal scheme. We are seeing that a great quantity can be formed by combining 4 numbers. That the quantity can be composed and taken apart. That no more than nine units of any category can stay loose, and with one more, it becomes a 10, a 100, or a 1000. We are presenting the child a whole, through which he can make an association of the numbers and achieve a mental synthesis. From this point, then, he himself can move on to the particulars.

Materials

1. The golden beads: nine unit beads, nine ten-bars, nine hundred-squares, and one thousand-cube.
2. The number cards: 1-9, 10-90, 100-900, and 1000.

EXERCISE:

- a. Arrange the materials on the table or mat with the number cards on the left side so that the 1000 is the farthest left at the top, then the row of 100s with the 100 at the top, and finally the units with the 1 at the top. Arrange the beads in corresponding order on the right side. The child may help this preparation.

- b. On the wooden tray, the teacher prepares a quantity in beads chosen from the displayed group; and asks the child to bring back with him (on the tray) the correct number symbols for this group of beads.
- c. When the child returns, the teacher controls error, laying the cards out first (units at the top, tens below, then hundreds, and thousands, flush left) and asking the child what he has brought. At first, the beads may be placed beside the cards for a further count and check.
- d. Then the teacher shows again the placement of the four categories in a stack, units on top and all flush left. . .and she asks:
 - Is this the number I asked you to bring?
 - Do you remember our magic trick to change this number?
 - Would you like to try it? (or she may demonstrate again)
 - Now, what is this? (units, tens, etc.)
 - What color is it?
 - How many zeros are hidden in this number?
- e. The color being of specific importance, this may also be a good time to ask the child to remember the color without looking.

EXERCISE:

- a. The materials are arranged as in the previous exercise.
- b. The game proceeds as before, except that this time the teacher prepares on the tray a quantity with the number cards. The child brings the corresponding beads.

NOTE: It is important each time, in both exercises, for the child to return his materials to the correct position after each completion.
- c. During this exercise, give quantities at times that have a zero in each of the categories: 1,405, 1,098 1,120 And ask:
 - What is missing here?
 - What is keeping its place?

Purpose

- DIRECT AIM:**
1. To familiarize the child with the way the numbers are written and the different categories of numbers.
 2. To give the child the possibility of writing big quantities with a limited number of symbols.
 3. To make the child realize that the zero keeps the place of a missing category: holds the position.
- INDIRECT AIM:** To gradually come to understand that only nine elements of one category can be loose.

Age 6

Control of Error Teacher

NUMERATION BEYOND TEN

We began our investigation of mathematics with the child by working within the limits of ten. Next we presented the decimal system. Now we turn to the work with the numbers 10-20 and 20-90. And because the child has already seen the general view of the whole number scheme, he can move towards this detail with interest and understanding.

THE SEGUIN BOARD 10-19

The Teen Boards

Material

1. A small box containing the colored bead-bar stair, with one each of the bars from 1-9.
2. A box containing golden ten-bars.
3. A box containing two rectangular slotted boards, color grey, each with 5 sections, and all except the last (on the second board) marked 10, in blue.
4. Wooden tablets, also grey with blue numbers, from 1-9 which fit onto the slots.

Presentation

of Quantities (Beads)

1. Begin by laying out a vertical row of 5 ten-bars, and indicating one ask: What is this?
2. Then say: With this ten and a one (placing the colored one-bead at the top of the first golden ten), we make 11. Let's count it.
3. "We have one ten and two, and the number becomes 12." Continue through 13, 14, and 15, asking the child to add the smaller bars if he is understanding quickly. After each number is formed, the child should have the opportunity to say the new word or count it.
4. If necessary for good comprehension, use the second and third period lesson after introducing the first five, 11-15:
Show me 13.
How is it formed?
What is this?
Will you count it?
5. Now introduce the next four numbers, first laying out the line of ten-bars and then adding the colored bead-bars with the child's help.
6. When the nine numbers are all laid out in beads, ask the child to count them in order. Then use the second and third periods.
Show me 15.
Show me 17.
What is this?
7. Now ask the child to count the numbers in order again, and then ask him to count them backwards. Finally, ask him to count them forwards and backwards without looking.

Purpose

DIRECT AIM: To learn the numeration from 10-19 forwards & backwards.

Presentation

of the Symbols

1. Place the two boards side by side, but spaced about 10 cm. apart, on the table and ask the child to lay out the wooden tablets in a row in a convenient place near the boards.
2. Now ask the child to look at the two boards and indicating the first ten, ask: What is this?
(and the second): What is this?
Yes, we have all tens on this board.
3. "When we formed 11 with the bars and beads, how did we make it? (With a 10 and a 1.)
Try to do the same thing here. (and if necessary the first time, show the child how to slip the wooden one tablet onto the right side of the 10)
Now ask: How is 11 formed?
With a 10 and a 1.
4. "Now how will we make the number 12?
Yes, a ten and a 2 make 12."
Will you make the 12 for us on the board.
5. Continue through 13, 14, 15, 16, 17, 18, and 19. Continue to say as the child places the tablets: "With a 10 and a 4, we make 14." etc. Also note during the completion of the numbers that the unit number, the tablet, always is placed over the zero.
6. When all the numbers from 11-19 are completed on the board, have the child read them forwards and backwards. If necessary, use the second and third period lesson for reinforcement while the tablets are in place.
7. Then remove the tablets and say:
Will you show me 15 on the board?
(If he puts it in the wrong position, show him the correct position, counting to that number to find it.)
Now count for us to the number you have made, beginning with the 11.
Repeat this exercise of having the child form the numbers and counting to them with several other numbers.
8. Now again take the tablets off, and form one of the numbers, asking: What is this? (16)
What is this?-----and here, finish with 19-----
9. "What do you think comes next? Our space here is empty.
What comes after 9?
How is the ten formed? NOTE: This information should be
After 19, comes 20. introduced only if the child
How is 20 written? asks about the blank section.
With a 2 and a 0."
10. "Now, if we want to go on after twenty, we have to use the other boards.

Purpose

DIRECT AIM: To learn the names and the sequence of the numbers from 11-19.

EXERCISE:

Matching the Quantities to the Symbols

1. Place on the table the two ten boards and the group of wooden tablets from 1-9. Also the ten-bars and the colored bead stair.
2. Ask the child to form the number 11 on the board.
3. Then say: Now will you make the 11 right here (indicating the left side) with the beads. How is it made?
With the 10 and the 1. Yes.
4. "Now will you finish making the numbers on the board, and for each one, made the same number beside it with the beads?"
5. The child can complete this exercise by himself. . . .to see if he can match the symbol and the quantity. When the teacher checks to control error, she might use the second and third periods: Show me 17. What is this?
Also a good time to note that after 19 comes 20, made of two tens.

Purpose

DIRECT AIM: To find out if the child knows the symbols and the quantities for the numbers from 11-19.

Control of Error Teacher

THE SEGUIN BOARD 10-90

The Ten Boards

Material

1. A small container of the golden unit beads.
2. A box containing golden tens.
3. A box containing the two rectangular slotted boards, color grey, each with 5 sections, and the first board numbered 10, 20, 30, 40, 50 and the second numbered 60, 70, 80, 90 and the last section blank. All numbers in blue.
4. Wooden tablets, also grey with blue numbers, from 1-9 which fit in the slots.

Presentation #1

1. Place the two boards, side by side with space between, on the table before the child, and close at hand the box of golden tens.
2. Indicating first the number on the board 10 ask:
What is this?
Then holding up a ten-bar, ask:
What is this?
Yes, this is one ten.
And place the golden bar to the left side of the symbol.
3. "Here we have two tens." (holding up the two golden tens)
"We call two tens twenty."
"Ten, twenty." (as this is said, place the golden tens, one at a time to the left of the number, beginning next to the board and counting out.)
4. "Here we have 3 tens.
We call the 3 tens thirty.
Now count with me---10, 20, 30.

5. Continue this pattern through 40, 50, 60, each time placing the ten bars to the left and counting by 10s. At 70, or perhaps before you may ask: (indicating the next number)
How many tens do you think we'll need to form this number?
Yes, seven tens.
We call 7 tens seventy.
6. Then 80, with child's help on number of tens and the counting by tens which he may be encouraged to do all along.
7. And this last number, how many tens do you think we'll need here?
Yes, nine. We call nine tens ninety.
(And proceed to count by tens)
8. Bring the child's attention to the similarity of the unit name and the ten: for example, six and sixty. Also note those that are different, specifically 20, 30, and 50.
9. Now have the child count by tens indicating the symbols with his finger. And then ask him to count the tens backwards.
10. Then beginning at 10 ask:
How many tens are here?
And how many tens in 20?
And how many tens in 30? until you reach
How many tens in 90?
11. Now indicate the empty section and say:
Here I have nothing.
What do you think comes next?
How many tens? 10. (place another 10 bars on the left here)
We already know that 10 tens is 100.
12. Now place the hundred square beside the empty section, saying:
Here is our hundred.
We'll place it here instead of the 10 bars of 10.
Now the tens are not loose, because this is something different, something new.
13. Remove the bead bars and ask: Show me 20.
Will you form it? (child places bars)
14. Third period: What is this? (indicating number)
How do you form it? (again child places beads next to number.)
Or teacher may form the quantity next to number and ask: What is it?

NOTE: Missing from the list of materials is the 100-square. Also it is needed in the next presentation

Presentation #2

The child now knows 1-19 and also the tens: 10, 20, 30, 40, etc. Now he learns to count to 100, one number at a time. In doing so, he is again working towards an understanding of the rule that one cannot have more than nine loose units of any category.

1. Place the ten boards again side by side on the table and also the wooden tablets from 1-9. Also on the table is the box of ten bars and the container of unit beads.
2. Ask the child first to count out ten unit beads, placing them one by one in a vertical line beside the number ten on the board. When he places the tenth bead say: You remember that we can't have more than nine of anything. What can we change these beads for?

- And place the ten bar beside the ten.
3. "What comes after 10?
Let's add one bead to the ten,
and it becomes 11. And we'll put the 1 (tablet) next to the 10.
10 and 1 makes 11."
 4. Continue adding unit beads and changing the unit tablets through
19. Then say:
If we add one more, what does it become? (20)
But we cannot have 10 loose beads.
We must change them for a ten.
Now we have two tens.
We have 20. (and the teacher moves the two bars into a new
position next to the symbol 20)
 5. Now add unit beads and change number tablets next to the two
ten bars saying:
Now 20 and 1 makes 21. . . .etc.
At 29 say:
With one more we will have ten, so we need another ten.
And three tens is 30. (moving the bars down next to 30)
 6. At this point the child may want to continue with the beads and
tablets on his own. Or if he seems to know the numeration well,
we might ask him to simply count between the tens, or write the
numbers in his notebook. . .noting the addition of another ten
at each new section that he reaches.
 7. At 90, however, tell the child that we must again count with the
unit beads and the tablets because something very important is
going to happen. Proceed to 99 and say:
Now what happens when we add the next bead?
What does it become?
We must exchange this group of ten units for another ten.
10 tens is a hundred.
But here we cannot have ten loose bars. We need the hundred
square. We have passed to something new.
99 and 1 unit takes us to something different.

Age 6

Purpose

DIRECT AIM: To make the child become skillfull in counting from
1-100.

NOTE: After the special study for the passage from 1-19, the real
difficulty is past. Now the numeration from 20-100 is
very quick. And soon the child will realize that only
nine numbers come into play. During this presentation, then,
it is important to allow the child to do the counting by
ones as quickly as he is able, counting aloud, writing
the numbers. Only from 90-99 is it particularly helpful
to use the full demonstration of the numeration one by one.

EXERCISE. . . FOR THE TEACHER

If the Seguin boards are not available, the same presentation can be given with other materials.

The numbers 11-19 can be presented with the RED AND BLUE RODS and the number cards, the simple first set of white cards with red numbers from 1-10.

Both the teens and the tens can be presented with the bead material and the number cards.

It is enough to use your mind.

NUMERATION FROM 1-100

The Chain of 100

The Chain of 1000

Materials

1. A chain of 100 beads, made of 10 golden bars linked together, each a 10-bar; the chain hanging on a hook.
2. A chain of 1000 beads, made of 100 golden 10-bars linked together. After every 10 bars, there is a ring from which the chain is hung on a series of hooks. (This must be done very carefully, and the child must be helped to see how to hang the chain orderly and how to take it down carefully. Draping it over his arm at the rings seems to work the best.)
3. Two envelopes containing paper arrows, one marked 100 and one marked 1000.
 - a. Envelope for 100 contains:
 - 9 tiny green arrows, numbered 1-9
 - 9 small blue arrows, numbered 10, 20, 30, 40, 50, 60, 70, 80, 90
 - 1 large red arrow numbered 100
 - b. Envelope for 1000 contains:
 - 9 tiny green arrows, 1-9
 - 9 small blue arrows, 10, 20, etc.
 - 9 large red arrows, 100, 200, etc.
 - 1 very large green arrow, 1000.

NOTE: These arrows should be displayed in order before each presentation. Perhaps displayed on a tray for ease of working and later the child, working alone, may find this a convenient way of keeping the arrows straight.

Presentation

The 100 Chain

1. Lay the 100 chain stretched out on the table and introduce it saying: This is the hundred chain. **How is it formed?**
Let's count how many tens are here? 1, 2, 3, . . .
How let's count this chain by tens. 10, 20, 30, ..
2. "Would you like to see why we call it the hundred chain?
Proceed to fold up the chain to form a hundred square.
"Look what we have made.
What does it look like?
Let's see if it is the same as our hundred square?
And at this point, place the square on top of the folded chain to show the equality. Point of IMPRESSION.
3. Now stretch the chain again to length and show the child how to place the tiny green arrows by each of the unit beads in the first ten, saying: We know that there are ten units in every ten. Let's mark them beginning with one.
4. When we reach 10, we place the blue 10 arrow beside that bead. Now we are ready to count the teens by moving the green arrows up one ten and we say:
What comes after 10? (11)
And what is 11?
It is ten and 1. (placing at this point the 1 on the first bead of the second ten)

And after 11?

What is 12?

It is 10 and 2.

Continue through the teens and place the blue 20 arrow on the 20th bead.

5. At this point the child may continue to move the green arrows to form 21, 22, 23, etc. Or he may count aloud, indicating the beads with his fingers and simply mark each ten with the blue arrows. (It is highly possible that he will not want to take the time to continue moving the arrows one number at a time.)
6. At the end, have the child place the red arrow indicating the 100th bead and say: Now let's put the hundred (square) at the end to show that it is the same, a hundred beads.
7. Have the child count by tens forwards and backwards, indicating the tens.
8. Also he may like to say all the numbers 1-100 backwards.
9. At this point, he may also like to write all the numbers in his notebook.
10. When the teacher comes to check the correct placement of arrows, a good opportunity to use the second and third period lessons, making them as difficult as the child is ready for:
Show me 70.
Show me 56.
Where would you look for it?
Use the arrows to find it, or count with your finger.
What is this? (50)
What is this? (88)

Purpose

DIRECT AIM: To learn the numeration from 1-100.
To help the child become skillful in counting by ones and by tens.

Moving Towards Understanding

At this point the format of the album changes. The change reflects several important ideas:

The aims, both direct and indirect, are presented first so that bearing in mind the idea of the overall purpose, we may proceed to implement it with the material and its presentation.

There is an approach to the liturgy. It is an effort to see clearly the progression of ideas presented verbally, and to understand the conciseness in their simplicity.

There is now an understanding that we must stir the interest of the elementary child and then, as soon as he is ready, let him continue the work of the presentation or the exercise. Thus we allow him the freedom to expand his interest and to abstract when he is able.

October 26, 1973

The 1000 Chain

Direct Aim: To have the child learn the numeration from 1-1000.
To help him become skillful in counting by ones,
by tens, and by hundreds.**

Materials

1. A chain of 1000 beads, made of 100 golden 10-bars linked together. After every 10 bars, there is a ring from which the chain is hung on a series of wall hooks.
2. An envelope marked 1000 containing:
9 tiny green arrows, 1-9
9 small blue arrows, 10-90
9 large red arrows, 100-900
1 very large green arrow, 1000
3. 10 golden hundred-squares and the 1000-cube.
4. A long corridor; perhaps a long felt strip on which the chain may be laid out.

Presentation

- | | |
|--|---|
| 1. Create the visual impression of 1000 by laying the chain on the table with the 100 bars lined up in rows of 10. | 1. This is the chain of 1000. Look at it carefully. |
| 2. Place a hundred square over each group of 10 bars. | 2. The chain is made of 10 hundreds. Let's count them. 900, and one more is 1000. |
| 3. Stack the squares and compare it to the 1000-cube. | 3. If we put the 100s together, what do we make? Now we are sure this is a chain of 1000. It is formed by 10 hundreds. |
| 4. Display the chain stretched to its length in a corridor or curved on a large table; present the arrows arranged in order. | 4. Now let's count this chain and place these arrows next to the correct beads. |
| 5. Show the position of the first 10 green arrows, the first two blue ten arrows. Have the child continue. He may count with arrows, simply aloud, or write the numbers. | 5. This is one.
This is ten.
Ten and one is 11.
After 19 comes 20. |
| 6. At 100, place the red arrow and then a hundred square beside it. | 6. We mark the hundred with a hundred square. |
| 7. Show the child how to move the tens to count the second hundred. Allow him to finish the exercise. | 7. Now we begin to count 101, 102.
This is 110. We mark it with the blue 10.
Will you go on now to count the chain of 1000. |

8. At the last bead the child places the large green 1000 arrow. Add the cube at the end.
 9. Exercise: count by 10s and 100s.
 10. Exercise: second and third period lessons.
8. Our chain is 1000 beads. This is 1000, too.
 9. Will you count the chain now by hundreds?
 10. Show me 200. This is a hard one. Find 417. Where will you look for it? What is this?

INDIRECT AIM: To begin an understanding of how the powers of 10 increase.

NOTE: With the 100 and 1000 chains, the child again sees the importance of the zero. This time, by observing the great difference in the length of the two chains, he observes that the addition of only one zero greatly increases the quantity. He gets a visual idea of the proportional increase with the powers of ten.

THE RED AND BLUE RODS
(numeration from 1 - 10)

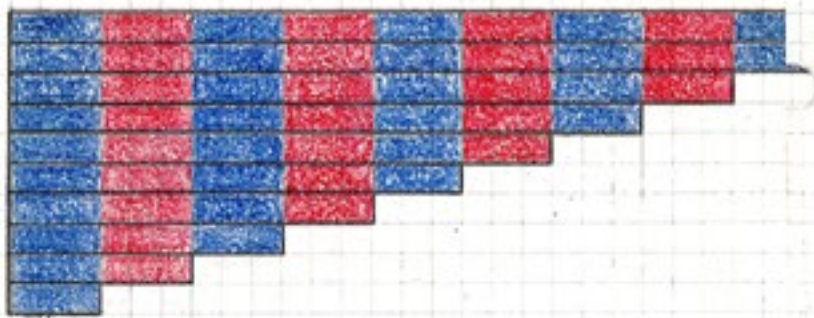
Materials

1. Ten rods representing each number from 1 thru 10, lengths differing from one to ten parts accordingly. The one-rod is blue, the two-rod is blue (part one) and red (part two); the others follow the pattern with alternating parts of blue and red.
NOTE: Each part is 10 centimeters, thus one decimeter. The 10-rod, then, is one meter.
Each side of a rod measures 2.5 centimeters, so that the volume ratio between the first and last rod is always 1:10.
2. Sandpaper numerals, durable hand-sized tablets of dark blue with the numerals from 0-9 raised in sandpaper (for the tactile exercise). These numerals are set slightly to the right side so that the tablet may be held easily with the left while the right hand fingers trace it.
3. Wall charts of a rectangular shape, a blue background and the sandpaper numerals again corresponding to the tablets.
4. A smaller set of numerals printed on cardboard and including the whole numeral 10, i.e., 1,2,3,4,5,6,7,8,9,10. No zero.

Presentation

NOTE: The child is, in this presentation and exercise, forming the numeral concept of the quantity for each number from one to ten. Thus, each rod represents a static quantity: an entity. And these ten quantities, along with their relationship to each other, form the base of our own system: the decimal system.

1. Begin with five rods (1-5), unless the child is ready, and then it is possible to begin with all 10. Pick them out of the whole group or bring them alone to the table or mat.
 2. Place them without order on the table.
 3. Say: This is one.
This is two.
Then count the two, using the one-rod as an indicator: one, two.
Then say: This is three.
One, two, three. (again indicated with the one-rod).
Continue in this manner until each number is introduced.
The child may join in the counting as each proof is made.
 4. Then say: Show me 1.
Can you count it?
Continue this second period through the numbers.
 5. Say: What is this?
Count it.
Continue this third period lesson through the numbers.
 6. Continue adding rods if the child knows this well.
- EXERCISE: a. Ask the child, as you pick up one of the rods, to "bring me the rod that comes before this one."
b. Say: What is it?
c. Then ask: Bring me the one that comes after this one.
d. Say: What is it?



The Red and Blue Number Rec

Purpose

- DIRECT AIM:** 1. To teach the idea of the quantity of the numbers from 1 to 10, and their names. Thus to acquire the concept of number, and how many units constitute each number from 1 to 10.
2. To memorize the succession of numbers from one to ten, and to learn those names of progression. (first, second, third, etc.)
- INDIRECT AIM:** 1. Preparation, unconscious, for the decimal system, and thus for the metric system as well.

Control of Error

Visual: Child sees that the colors either line up or they don't. Also views the stairstep periphery.

Age 6

SANDPAPER NUMERALS

Materials

1. Small wooden box where the numeral cards are placed, in either of two sections.
2. Dark blue numeral tablets (cards), hand-sized, for the numerals from 0 - 9 which are raised in sandpaper and placed slightly to the right of the center of the card.
3. Wall charts of light blue, rectangular-shaped, with corresponding raised sandpaper numerals.

Presentation

1. Decide how many numerals are to be presented. (perhaps start with five, 1-5)
2. Lift them from the box, containing only those to be introduced, one by one, and say: This is zero.
I will show you how it is written.....
And as you say this, trace with the fingers the motion of the written numeral along the sandpaper.
3. Then say: Try it.
Be sure that he has seen it and imitated the motion correctly.
4. Second Period Lesson: Say: Show me zero.
How do you write it?.....
Inviting the child to trace the sandpaper, or if inclined, to write it on the chalkboard or with pencil and paper.
5. Third Period: Say: What is this?
How do you write it?
6. Use the three-period lesson with the wall charts and the numeral cards. Ex.: Pick up one card and say: Find this one. (on the wall chart)

Purpose

1. To give the child the corresponding symbol for the quantity.
 2. Preparation for writing numerals.
- NOTE:** Both are direct aims.

MATCHING THE NUMERALS WITH THE RODS

Materials

1. The red and blue rods.
2. Sandpaper numerals, on the dark blue tablets.
3. Smaller white cards (cardboard) with red numbers on them, the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10.

NOTE: The sandpaper tablets prepare the mechanical process of writing the numerals. The white cards, now introduced in this presentation, introduce the value of the number---which is a mental process of learning.

Presentation

1. With the white cards in hand, lay the 10 alone on the table and say: This is ten. (child has not seen this number before.) And then: Ten is made with a one and a zero.
2. Then place all the cards in random order on the table, and say: Notice that this card is twice as big as the others, (indicating the 10 card) and we can place another number on this part (first half) or this part (second).

Demonstrate this combination of the ten with another number with several of the cards.

3. Reveal the red and blue stair (rods), already prepared in order on the same table.
4. Give the number cards to the children one at a time and say:
This is one.
Will you put this number on the correct rod?
(If children hesitate, or it seems necessary, place the first one correctly to the left of the correct rod yourself)
Continue until all rods are identified with a card.

- EXERCISE: a. Take a rod and the child to find the corresponding card (number). Both materials in random order.
- b. Take a number (card) and ask the child to find the corresponding rod.
- c. Use many children in exercises as most elementary presentations are given in groups of at least two. All can participate.

- EXERCISE: a. Separate the cards and the rods some distance apart in the room, perhaps on two tables at different ends.
- b. Give the child a rod, ask him to look at it carefully, and then go and bring back the right number card. On return, say: What number is it?
Will you count it to make sure?
- c. Repeat, but begin with the number card.

Direct Aim/Purpose

1. To introduce the number value, that which corresponds to the previously formulated idea of quantity.
2. To give the idea of the hierarchy of numbers. (10, 11, 20, etc.)

Age 6

IDEA OF ADDITION AND SUBTRACTION

Materials

1. The red and blue rods.

Presentation

1. With the red and blue rods arranged correctly in the stair, say: What is this? (holding up the ten rod)
And then: Let's see how we can form a ten with the other rods. We can form many tens.
2. "First let's put the one with the nine. . .and here is ten. Now let's put the two with the eight. . .and here is ten. Now we put the three with the seven. . .and here is ten. Now we put the four with the six. . .and here is another ten." Finally, having placed each of these rods together with another, we can say: We have formed ten in four ways.
3. Then, pointing to the five-rod, we comment:
"But we have a five left over.
What do we need to make this a ten?"
And then we show how the five can be turned over to fill the empty five and say: Two 5's make a 10!!
4. Then we make the opposite demonstration, saying:
"If I take this one away from the ten, I have nine.
Will you count it? (the remainder)
And proceed with the other minus operations thusly.
Child counts the remainder each time, and if it looks necessary, we can replace and recount the whole ten first, and the two parts afterward.
5. Finish with the return of the five to its original position.

Purpose

- DIRECT AIM: 1. To find out if the child has understood the quantities from 1 thru 10.
- INDIRECT AIM: 1. To give an introduction to the ideas of addition and subtraction.

THE SPINDLES

(There is an important difference between the rods and the spindles. With the rods, the quantity is one whole. With the spindles, they are separate parts that make the quantity. The child himself must unite the quantity.)

Materials

1. Two boxes, each with divisions that mark off five compartments. Above each compartment is written the number, from 0 - 9.
2. 45 wooden spindles. (This is error control--all are gone from the container in which we present the spindles when the presentation is finished)