EXPLORATION OF THE QUADRILATERALS

The quadrilaterals of reality are six in number. The child learns this from his work with the plane figures of the geometry cabinet and with the cardboard series of picture cards of the figures. We list the six quadrilaterals from the general to the specific, according to the genealogy of the square. New math also follows this order.

We begin our exploration of the plane figures, as we did with the triangles, from the humblest beginnings: 1) common quadrilateral or trapezium, 2) trapezoid, 3) parallelogram, 4) rectangle, 5) rhombus, 6) square.

Presentation (Material the same: the box of sticks and supplies)

1. Ask the child to choose four sticks and unite them in any way. His only qualification is that he shall choose four, thus a figure with four sides will result. If he should choose four that are the same, we still proceed with the common quadrilateral.

1. This is a quadrilateral.
Let's count the sides. . .1,2,3,4,.
This is a common quadrilateral.
Common means "any quadrilateral."

2. Ask the child to construct another quadrilateral with the same four sticks. Then, using another stick to establish two sides as parallel, shift the position of the four sticks so that there is one pair of parallel sides.

2. It is enough to make one pair of these sides parallel in order to create another quadrilateral.
This is a trapezoid.
A common quadrilateral having at least one pair of parallel sides is a trapezoid.

3. LEAVE THE TWO CONSTRUCTED QUADRILATERALS ON THE MAT. Then ask the child to choose two pairs of sticks. Unite one of each pair in two angles. Then put the two angles together with opposing equal sides. Show as a parallelogram.

3. This quadrilateral has two pairs of parallel sides.
This is the common parallelogram.

4. Construct the same two angles with the same two pairs of sticks. Show the same parallelogram and then push the figure to a rectangle.
Measure the angles with the measuring angle.

4. Now it is not enough to have two pairs of parallel sides.
Here we have four equal angles. And those four equal angles are right angles. Let's measure them.
This quadrilateral is called a rectangle.

5. LEAVE THE FOUR CONSTRUCTED FIGURES ON THE MAT. Ask the child to construct a quadrilateral with four equal sticks.

5. With four equal sides we form a rhombus.

6. DISPLAY NOW ALL THE CONSTRUCTED FIGURES IN A DIAGONAL FROM THE LEFT TO RIGHT. And ask the child to take another four sticks---the same ones used to construct the rhombus. Push the figure to a square.

6. This figure has four equal sides and four equal angles---right angles.
Let's measure those angles with the measuring angle.
This is a square.

7. Review the nomenclature of the figures constructed. Use the second and third period lessons. Then note the special position of the square.

NOTE: Later we will say that the square is the intersection of the sets of the rhombuses and the rectangles.

7. What is this? A Square.
How are the sides?
And how are the angles?
So the square is the little son of the rectangle and the rhombus.
In geometry the son takes the good qualities of both parents.

REGULAR AND IRREGULAR POLYGONS. . .
Presentation: From Irregular to Regular. . .

5. The child writes labels (as in Series #1) for the four hexagons:
   a) On Hexagon #1, he places the label "a non-equilateral hexagon"
   b) On Hexagon #2, he places the label "an equiangular hexagon"
   c) On Hexagon #3, he places the label "an equilateral hexagon"
   d) THEN he places on Hexagon #1 another label: "a non-equiangular hexagon."
   e) FINALLY he takes the labels from #2 and #3 and shows them on #4 Hexagon.

NOTE: all these read "polygon."

6. Analyze the labels as shown now, two on Hexagon #1 and two on Hexagon #2.

Then identify #2 with one label from Hexagon #1 and one label from Hexagon #4 so that it is identified with one positive and one negative label; repeat for Hexagon #3. Analyze.

6. Here I have four polygons. The first one has two negative qualities. The last one has the same characteristics as the first, but they are positive. NOW I can identify the second polygon with one negative characteristic: a non-equilateral polygon, and one positive characteristic: an equiangular polygon. We have taken a bad characteristic from polygon number 1 and a good one from polygon number 4. I can do the same thing to identify polygon #3. Which labels will I use? So the first figure is bad---far from perfection; and the second, the last one, is good---perfect.

7. Remove the series #1 labels and match the four labels of series #2 to the displayed constructed figures.

8. Define the regular polygon, recalling the intuition experience of the first presentation regarding equality of angles and sides.

8. Which of these figures has two positive qualities together? When we say that a polygon is equilateral AND equiangular, we say it is a REGULAR POLYGON.

9. Remove series #2 labels and introduce series #3 (the child writing these each time). Put the two labels at the top of the mat and group the four figures under the appropriate one.

9. Now we can divide these figures into two groups: Regular and Irregular Polygons. Which are the irregular polygons? Let's write a label to identify that group. And this polygon is regular. Let's write another "Regular Polygon."

10. Show a final arrangement of the figures:

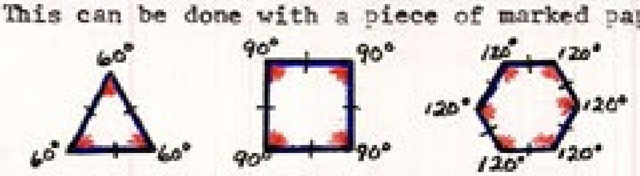
We are showing the movement toward perfection.

#2
Irregular Polygons
#1 #3

#4
Regular Polygon

Here we verbalize this movement from the most irregular to the most regular of the polygons shown. And we conclude that we need the positive characteristics of both #2 and #3 to achieve #4.

11. ACTIVITY: On the original drawings of the figures which the child has made, he has already marked the equal angles and the equal sides in colors. Invite him now to measure the angles and write the degrees. Then to measure (with a ruler if he has had a lesson in measuring in centimeters) the sides. This can be done with a piece of marked paper to prove equality.



12. CONCLUDE: These are all regular polygons because they are, at the same time, equiangular and equilateral.

AGE: Presentation #1: 8
Presentation #2: 9

Presentation #3: Nomenclature of the Square and the Triangle

Material
1. The plane insets from the cabinet.
2. The Classified Nomenclature

Presentation

Take from the cabinet drawer (#1, #2, or #3) the equilateral triangle and the square. Review the familiar definition and then redefine, using the polygon definition, the second in the Classified Nomenclature for the two figures.

When we first described this triangle precisely, we defined it in terms of its sides: The triangle with three equal sides is called an equilateral triangle.
Now we can define it in a new way:
A regular polygon having three sides is an equilateral triangle.
We first defined the square, when we included it in the set of parallelograms as a parallelogram having equal sides and equal angles.
Now we can define it according to our knowledge of polygons: A regular polygon having four sides is called a square.

CONCLUDING ACTIVITY: Let's Construct the Polygons

Here the child reproduces the experiences he has done with the wooden sticks in a new way. In his notebook he constructs the polygons from the triangle to the decagon.

Material
1. Different colored straws.
2. Long needles with blunt points. (Bodkin needles)
3. A ball of yarn.
4. Scissors.

Presentation: Introduction of the materials and the mode of the activity

1. First the child must decide which figure he wants to construct: then whether that figure will be constructed according to sides or angles: then what kind of sides or angles. As an example, we shall decide to construct the equilateral triangle first.

2. For this construction, we take three straws of the same color since our sides will be equal. Then we establish the length of one side and cut the straw to that length. (This is my unit) Then, measuring with the first straw, we cut the second two to the exact corresponding lengths.

3. Using the needle threaded with the yarn, we thread the three segments together and tie the loose ends.

4. Another figure: the parallelogram. We need two pairs of equal sides, so we choose two pairs of straws in different colors, first cutting one pair equal and then cutting the second pair equal of another length. Then we must thread them, alternately, first one of one pair and then one of the other. When the thread is then tied, it is obvious that we must use a diagonal to stablize the figure or it will become a rectangle. So we fix the parallelogram and cut a diagonal OR a third color that will correspond to the parallelogram we have made. In order to string this, it is necessary that WE HAVE LEFT ONE OF THE TIED ENDS LONG ENOUGH SO THE DIAGONAL CAN BE THREADED ON IT. We thread the diagonal and tie it at the opposite vertex.

5. The child pastes the figure to his paper.

NOTE: In order to do this work, the child must have thorougly understood the character of the polygons. Later he can cut straws of specific lengths indicated by prepared commands: after he is familiar with a system of measuring lines.

THE CIRCLE

I. Nomenclature of the Circle and Its Properties
II. Mutual Relationship of a Straight Line and a Circle.
III. Relationship of the Positions of Two Circles.

PART I: NOMENCLATURE OF THE CIRCLE AND ITS PROPERTIES

The child already knows the circle---that it has no beginning and no end, and that it is a closed curved region. He has had additional reinforcement of the idea of the circle in Drawer #5, seeing smallest and largest circles in graduation. But these qualities of small and large are not enough. We must now reconsider the figure.

Material

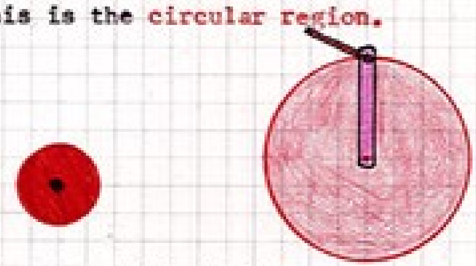
1. The plane and the box of sticks.
2. The metal insets (fractions) of circles.
3. A red pencil. (always present in a presentation of the new)

Presentation

1. Choose any stick (a longer one) from the box and fix it at one end (near center) with a red-headed nail. This is the center of the circle. Then, inserting a red pencil in the last hole, draw a red circle on the plane.

2. Show the plane inset circle. Compare. Then color the circle on the plane red.

1. This is a circle.
This is the circular region.

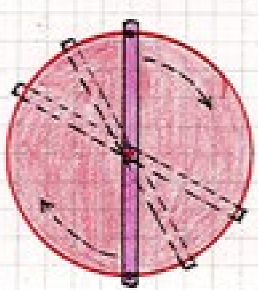


These are both circles.
The only difference is the size of each.
The circle on the plane is that part of the plane colored red.

3. In our construction we have already used the center and the radius, but we have not given the name. Now we give the NOMENCLATURE.

3. The circumference is this closed curved line which encloses the red part of the plane. . .but we must define it also in reference to the distance. . .
The pink stick is the radius.
The radius represents the distance between the center and each of the points on the closed curved line---that is, the circumference.
That means that the circumference has the same distance between it and the center at every point.
The center is an interior point which is equidistant from all points on the circumference.
To define the circumference, we need the center---to define the center, we must refer to the circumference.

4. Take another stick of the same length and fix it on the plane, taking the red nail out and fixing the two together. Then move the second stick to form a straight angle with the first, showing the diameter and rotate that diameter.



4. This is the diameter.
All of these are diameters of this circle.
The diameter is the line segment that unites two points on the circumference.
Remember that we removed the first stick in order to fix the second one on the plane with the same center.
So this line segment, the diameter, passes through the center.
Notice that the diameter is formed of two equal sticks.
SO THE DIAMETER IS ALWAYS EQUAL TO 2 RADII.

THE CIRCLE. . .
Presentation: Nomenclature. . .

5. Take a pencil of a different color and, inserting it in the same hole, draw along only a short distance showing an arc on the previous line. (The pencil goes through both sticks.)

5. We have gone from here to this point. This is an arc.
But this other part of the circle is also an arc.
The blue part of the circumference is one arc; and the red part of the circumference is another arc.
An arc is a part of the circumference--- it is both parts of the circumference.

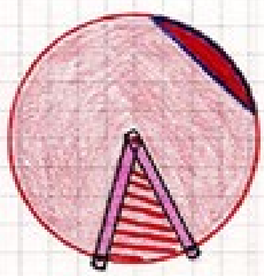
6. Unite with a colored pencil mark the two end points of the drawn arc.

6. We call this a cord.

7. Show the diameter again, introducing the semi-circle.

7. If all this is the circumference and the diameter divides it into two parts, we call each part of the circumference a semi-circumferende. . .
And each of the two parts of the circle a sem-circle.

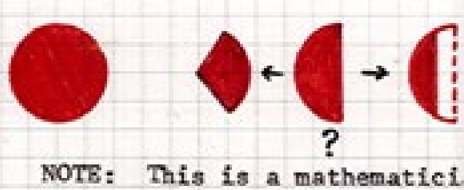
8. Analyze the parts regarding the surface: a) Color the segment in a darker red.
b) Fix two rays to show the sector and color it in stripes.



8. That part of the circle enclosed between the cord and the arc is called the segment of the circle. We will color it in a darker red.
The cord divides the circle into two parts and EACH PART IS A SEGMENT.
The part of the circle between an arc and two rays is the sector of a circle.
Not only this part that we stripe with color, but also ALL THE OTHER PART OF THE CIRCLE IS A SECTOR.

9. Give a careful three-period lesson, using the Classified Nomenclature and the plane/sticks construction AND

10. A GAME: Show the metal insets of the circle 1/1, 1/3, and 1/2. Then from another frame, take the inset showing the circle segment.



NOTE: This is a mathematicians dispute: Is the semi-circle the limit of the set of sectors and the set of segments? IF NOT, then we must better define the characteristics of the sectors and the segments.

What is this?. . .a circle, a sector of a circle, a semi-circle, a segment of a circle.
Let's define each carefully:
What is a sector? It is that part of a circle enclosed between 2 radii and an arc.
Is the semi-circle also a sector? I don't know.
What is the segment? It is that part of the circle enclosed between a cord and the arc.
Is the semi-circle a segment? Its cord runs through the center.
THEN THIS SECTOR (the semi-circle) IS A SEGMENT!!!!

SO. . .The sector is that part of the circle between the arc and the two radii which do not pass through the center of the circle.

AND. . The segment of the circle is that part of the circle between the arc and the cord which must not pass through the center of the circle.

THEN. .Now that we have specified that the radii must not pass through the center to form a sector and that the cord must not pass through the center to form a segment, the semi-circle CAN BE NEITHER A SEGMENT NOR A SECTOR.

PART II: Relationship Between the Position of the Straight Line and the Circle

Material

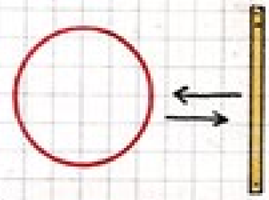
1. The box of sticks and the supplies.
2. A big wooden circumference, red.

These are the two elements involved: the circumference and a stick which represents the infinite line.

Presentation: FIRST LEVEL (without the radius)

CASE #1: External

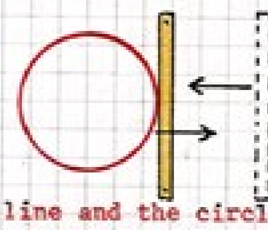
Here we show the line external to the circle or vice versa. Our relationship is between the circumference and the line both of which are external to the other, so we can move the stick towards the circle, stopping before we reach the circumference, OR we can move the circle towards the line:



One is external to the other. They are external because they do not touch---they have no point in common. The straight line is external in relation to the circle; the circle is external in relation to the line.

CASE #2: Tangent

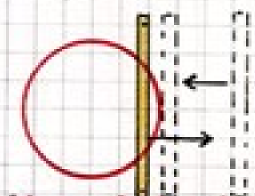
We show the line external to the circle, and then, verbalizing the movement. . . external. . .external. . .external. . . we move the line towards the circle until we hit tangent. OR we move the circle. . .



The line and the circle are tangent because they touch. They have ONE point in common.

CASE #3: Secant

We repeat the previous movement, verbalizing through externals, then tangent and, finally as the line moves into the circle, secant. OR we move the circle:

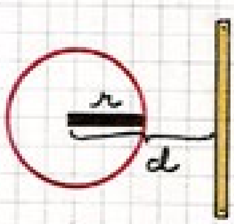


The line and the circle are secant because they touch---they have TWO points in common.

Presentation: SECOND LEVEL (with the radius)

CASE #1: External

Fix the stick in the circumference which shows the radius of that circle, using the MEASURING ANGLE then to measure the various distances.



$d > r$

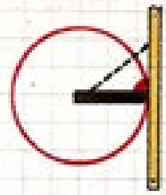
I know that the straight line is external to the circumference and that the circumference is external to the line because they do not have a point in common.
Consider now the distance between the center of the circle and the straight line. . .IS IT THE SAME LENGTH, GREATER THAN, OR LESS THAN THE LENGTH OF THE RADIUS?
The position of the straight line is external when the distance between the straight line and the center of the circle is greater than the radius.

By calling the radius r
and the distance d,
I can write this:

$d > r$

THE CIRCLE. . .
Presentation: Position of a Straight Line and a Circle: Second Level. . .

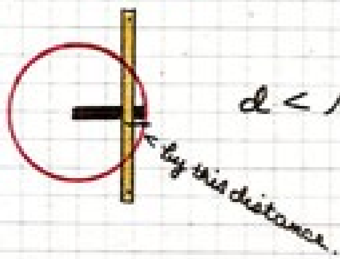
CASE #2: Tangent



$d = r$

We know that the line and the circle have one point in common.
What is the distance between the straight line and the center of the circle? EQUAL TO, GREATER THAN OR SMALLER THAN THE RADIUS.
It is equal, SO the line tangent to the circle shows $d = r$

CASE #3: Secant



$d < r$

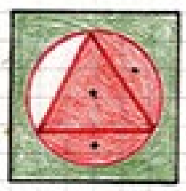
We know that the line and the circle here have two points in common. They are secant to each other.
How can we express the distance between the straight line and the center of the circle?
When the line is secant to the circle,

How much less than the radius?
The distance from the point of intersection of the radius and the line to the arc.

Note: on materials used for Part I: Nomenclature, Game #10. The inset showing the segment of the circle is one of the three segments shown in the additional inset below:

The Inscribed Equilateral Triangle Inset

diameter of circle = 10 cm.
so height of triangle = 10 cm.

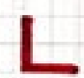


The result is 3 equal segments or, as shown here, with one segment removed, the cord defines two segments shown in the inset: both the white part and the red part are circle segments.

PART III: The Relationship of the Positions of Two Circles

Material

1. Two wooden circumferences of two different diameters: the large red circumference used in the presentation of the circle and the line, and a smaller black circumference. (These are embroidery hoops)
2. The box of sticks.
3. Measuring angles: a) the familiar measuring angle
   b) a simpler measuring angle, formed of two perpendiculars, in red.

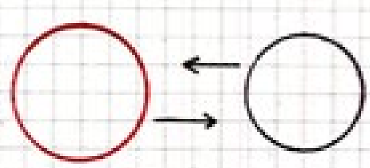


NOTE: The presentations are made on two levels for each of six cases.

Presentation: FIRST LEVEL (without the radius): Sensorial experiences

CASE #1: External

Show the two circumferences on the plane. Move either one towards the other.

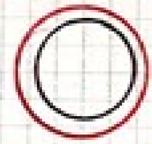


How are these two circles?
One is external to the other.
They have no point in common.

THE CIRCLE. . .
Part III: Two Circumferences. . .
Presentation: FIRST LEVEL. . .

CASE #2: Internal

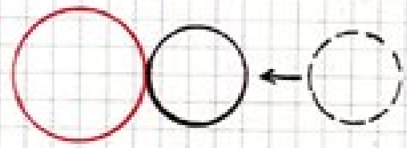
We place the smaller circumference within the larger, making sure that there is no point in common.



One circumference is internal to the other. The two have no point in common.

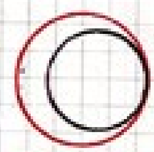
CASE #3: Externally Tangent

Here one must be outside the other, and they are tangent. We move one towards the other until this tangent occurs.



external. . .external. . .external. . . tangent.
The two circumferences have one point in common. They are externally tangent.

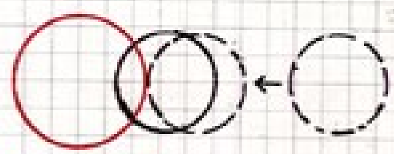
CASE #4: Internally Tangent



The smaller circumference is internal to the larger. The circles are tangent: they have one point in common. They are internally tangent.

CASE #5: Secant Circles

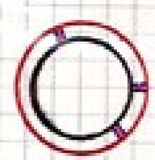
We move the circumferences toward each other, moving only one or the other, from an external position, past the tangent, finally to secant.



external. . .external. . .external. . . tangent. . .now neither external nor internal
The circles have two points in common. They are secant circles.

CASE #6: Concentric circles

We show one circle within the other. There is no point in common. Both must have the same center. With a short stick from the box, show that the distance between the two circles is the same at every point.

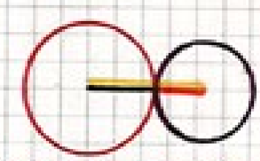


I can say that both the circles have the same center because there is the same distance between them at every point.
We have formed the ring.
These are concentric circles.

Presentation: SECOND LEVEL

We must begin this work by discovering the stick which will represent the radii for the two circumferences. Now we have as a point of reference the distance between the two centers of the circles.

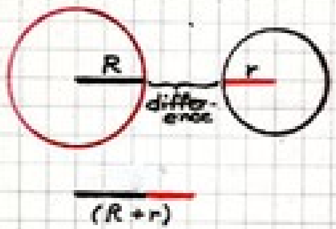
REVIEW THE NOMENCLATURE OF THE CIRCLES and then establish the two centers. Show a longer stick between the radii to indication a prolongation of the two, thus showing the distance between the two centers.



Since we will consider the distance between the two centers, we must place the radii so that one is the prolongation of the other. That stick represents the distance from this center to this center.

THE CIRCLE. . .
Part III: Two Circumferences. . .
Presentation: SECOND LEVEL. . .

CASE #1: External

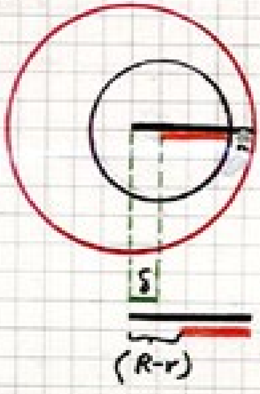


This is our plane.
These two circles are co-planer.
How are these circles? External.
Look at the distance between the two circles
How is it? Is it equal to, larger than or smaller than the sum of the two radii?
It is larger. . .by this part: the distance between the two circles.
I can make the proof by putting the two radii sticks together to show the lesser distance they make in relationship to that shown above.

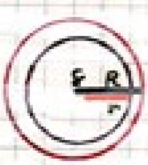
Develop the formula, using square of paper R, r and δ (delta) to indicate respectively the longer radius, the shorter radius and the distance between the two centers.

SO with δ and $(R+r)$
What is the sign after δ ?
THEN $\delta > (R+r)$
AND $(R+r) < \delta$

CASE #2: Internal
Here we are using as a reference the difference between the two radii (R - r).
We discover that [redacted] the distance between the two centers is less than that difference.
A LARGER CHART, AS FOLLOWS, IS NECESSARY TO SHOW THIS.



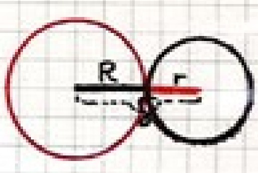
Tell me when two circles are internal: when one is inside the other and they have no point in common.
Is the distance between the centers of the circles (here shown as the distance between the radii at the center or the distance between the circumferences) larger than, equal to, or smaller than the DIFFERENCE between the two radii? (R - r)



It is less by the distance between the two circles (x).

So $\delta < (R-r)$
and $(R-r) > \delta$

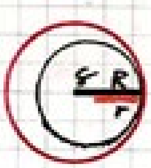
CASE #3: Externally tangent



If I place one finger on each of the two centers, the distance between my fingers is the distance between the two centers.
Is it larger than, equal to, or smaller than the sum of the two radii? Equal.

So $\delta = (R+r)$
$(R+r) = \delta$

CASE #4: Internally tangent



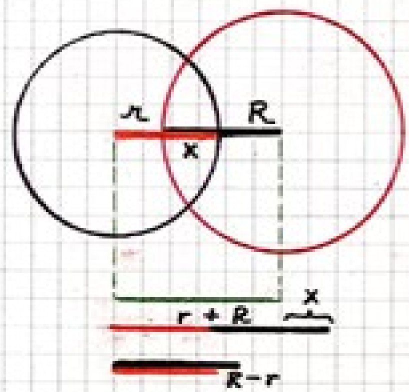
When the circles are internally tangent, one is internal to the other and they have one point in common.
Here we compare the distance between the centers with the difference between the radii.
We can observe that they are equal.

So $\delta = (R-r)$
$(R-r) = \delta$

THE CIRCLE. . .
Part III: Two Circumferences. . .
Presentation: SECOND LEVEL. . .

CASE #5: Secant Circles

Here again a large chart demonstrating this case is helpful:

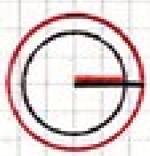


When two circles are secant, they are neither external nor internal; they have two points in common.
On the chart we can see below the circles both the sum of the radii and the difference between the radii.
The green line shows the distance between the centers.
Look at that distance.
Is it larger than, equal to or smaller than the sum of the radii?
The distance between the centers of the circles is less than the sum of the radii.. . .
Is that distance larger than, equal to or smaller than the difference between the two radii?
. . ...and greater than the difference between the radii.

THEN : $\delta < (R+r)$
$(R+r) > \delta$

AND : $\delta > (R-r)$
$(R-r) < \delta$

CASE #6: Concentric Circles



Where is the center of the black circle?
Where is the center of the red circle?
Then we can indicate both those centers with one finger.
What is the distance between those two centers?
There is none.
When circles are concentric, there is no distance between the centers.

AIMS OF THE CIRCLE WORK

Direct Aim: Knowledge of the circle and its parts.

Indirect Preparation: For the calculation of area of the circle and its parts.
For the study of inscribed and circumscribed polygons.

AGES FOR THE PRESENTATION OF THE CLASSIFIED NOMENCLATURE OF THE CIRCLE: 8 - 9 years

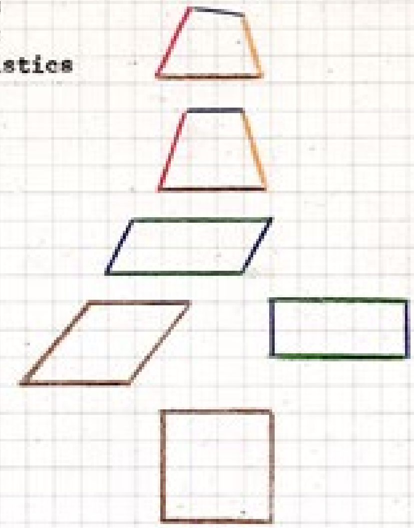
Relationship of the circle and the straight line: First Level - 8½
Second Level - 9

Nomenclature of the circle and parts: First Level - 8
Second Level - 9

Relationship of the circle in relation to another circle: First Level - 8½
Second Level - 9+

EXPLORATION OF THE QUADRILATERALS. . .
Presentation. . .

8. Rearrange the figures in the following way: Review the characteristics of each figure.



four sides.

at least one pair of parallel sides; thus the name.

two pairs of parallel sides; thus the name parallelogram.

rhombus: four equal sides.
rectangle: four equal angles.

four equal sides. . .
four equal angles. . .
THE SQUARE.

9. Now we can describe these figures according to the sets within which they fall. For example: all the figures are common quadrilaterals. All those figures shown below the trapezoid are trapezoids. The clarification of the relationship between the last three figures is especially important.

9. Is the rectangle a rhombus?
Is the square a rhombus?
Is the rhombus a rectangle?
Is the rhombus a square?
Is the square a rhombus?

10. Then reverse the direction of the exploration, showing the limits of the sets.

10. The trapezoid is a common quadrilateral, but the common quadrilateral is not a trapezoid.
The square is a rectangle, but the rectangle is not a square.

GAME: Here, by requesting a particular set, we are enclosing within the limit a particular group of figures.
NOTE: We have applied new math without mentioning sets. We have laid another part of the foundation for the great work ahead.

Give me the quadrilaterals.
Give me the trapezoids.
Give me the parallelograms.
Give me the rhombuses.
Give me the rectangles.
Give me the squares.

Presentation #2: The Diagonal

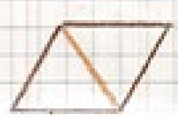
In the work with his hands, the child realizes that the figures he is forming are not stable. Thus he can make a trapezoid from a common quadrilateral by shifting the united sticks. This is contrary to his experience with the triangle construction. So we help him make this observation.

1. Show the triangles from the three boxes of constructive triangles.

1. These are called constructive triangles because they construct other figures.
They construct all the figures of reality.
Because all the figures of reality are composed of triangles.
Remember the quadrilaterals we constructed with these triangles?

2. Show the rhombus as an unstable figure. Then, with another stick, unite two vertices.

2. When we constructed the rhombus, it was not stable.
Now it is stable.
We have formed two triangles.



EXPLORATION OF THE QUADRILATERALS. . .
Presentation: The Diagonal. . .

3. Use a stick from the Pythagorean box to make the square a stable figure. Then stabilize each of the quadrilaterals with a diagonal.

4. Give the name.

4. This line segment which joins opposite vertices is called the diagonal.

5. Using a constructed triangle, show that there is no diagonal for the triangle.

5. I want now to unite two vertices of the triangle to show the diagonal. "I have forgotten to tell you about the diagonal of the triangle." Maybe I can unite these two vertices. But I already have a stick there. It is the side. Let's try these two. This triangle has no diagonal. The diagonal is only present in figures beginning with the quadrilateral.

6. Note that two triangles have been formed in the figures with each diagonal.

6. Into how many triangles have I divided each of these figures? I have created two in each with the diagonal. In working with the constructive triangles, we used two triangles to construct a quadrilateral.

Presentation #3: The Nomenclature of the Quadrilaterals

Material: the same, with the reading labels to identify the six quadrilaterals

1. Using the six quadrilaterals of reality which have been constructed by the child according to those constructed in presentation #1, show them displayed horizontally on the mat thusly: common quadrilateral, trapezoid, parallelogram, rectangle, rhombus, square.

2. Now the child uses the plane inset figures, matching each of the constructed quadrilaterals:
    a) From the Drawer #3, The Quadrilaterals, he uses the square and the rectangle.
    b) From the Drawer #6, Various Other Figures, he uses the rhombus and the trapezoid.
    d) From the box of supplementary figures, he uses the cardboard figures of the parallelogram and the common quadrilateral. (We do not yet consider the various trapezoids in this box.)

3. Having matched each of the constructed quadrilaterals with one of the plane figures on the mat, the child now matches each with the corresponding reading label.

4. ACTIVITY: Using the plane figures as pattern, the child draws the six quadrilaterals of reality and names them on his paper.

5. Now the child uses a stick to make each of the constructed figures rigid; a neutral stick required for the square and other sticks are found for the diagonals of the other figures.

5. Before we can work with these constructed quadrilaterals, we must use the sticks to make diagonals for each of them in order to fix the figure. Without that diagonal, we would find our figures unstable and constantly changing shape.

6. Our examination of the nomenclature now begins, first considering the common quadrilateral and then moving along our arrangement of quadrilaterals displayed.

7. Review again the visual examination of the altitudes of each of the six quadrilaterals, naming the kinds of altitudes in each.

NOTE: There is a special construction of the parallelogram in which both the altitudes are exterior: very long pair of parallel sides and another pair of very short sides.

7. In the classic trapezoid, with the major side as the base, all the altitudes are interior.
With the minor side as the base, all the altitudes are exterior.
In the parallelogram, the altitudes alternate: interior, exterior, interior, exterior.
In the rectangle, the altitudes fall along the sides (the limit) or they are interior altitudes.
In the rhombus we discover alternating altitudes: exterior, interior, exterior, interior.
In the square, the altitudes are interior or they fall along the side (the limit.)

8. Examine the diagonals formed in each of the figures constructed. The child then fixes the second possible diagonal in each quardrilateral. If he doesn't have the correct sticks, he makes one out of cardboard, fixing one end and then cutting at the other. He is discovering that diagonals intersect forming a cross AND that sometimes diagonals are equal (square and rectangle) and sometimes they are not.

8. We know that the diagonal is a line segment uniting two opposite vertices.
What does the diagonal form in each of our quadrilaterals.
We have in each two triangles, the constructors of the quadrilateral.
How many diagonals are possible in the quadrilateral?
We have another pair of opposite vertices . . .therefore, we can establish between these two a diagonal.

EXERCISE: Here we present the form entitled "Geometry---Sum of the Interior Angles of a Polygon." The first division, questions 1-3 represent the first level of work which he is ready to do now in connection with both the triangles and the quadrilaterals. Because there are three rows of spaces for each of the questions, he can do the exploration in relation to three of the triangles and three of the quadrilaterals. (SEE FORM)

THE TRAPEZOID

Materials

1. The box of sticks, and construction supplies.
2. All the trapezoids in the material.

Presentation

1. Separate the constructed scalene trapezoid from the group of quadrilaterals used in the previous presentations, and put the other quadrilaterals aside. DISPLAY on the mat this constructed figure with the isosceles trapezoid from the cabinet and the three trapezoids from the box of supplementary figures.

2. Using the constructed scalene trapezoid and the scalene trapezoid from the supplementary figure box, introduce the scalene trapezoid and place the two together on the mat.

The trapezoid has different shapes according to its sides and the position of its angles.
It has a minor base, a major base, an oblique side, another oblique side.
This is a scalene trapezoid.
So far we have used scalene to describe only one case---remember, when all the sides are different.
Here scalene refers to the oblique sides.
The trapezoid is scalene because those two sides which are not bases are of different lengths.

EXPLORATION OF THE QUADRILATERALS. . .
PResentation: The Nomenclature. . .

A. The common quadrilateral: Begin bv reviewing the familiar nomenclature. NOTE: this figure has no base and no altitude---we can say it rests on the longest side, but we can never call it the base.

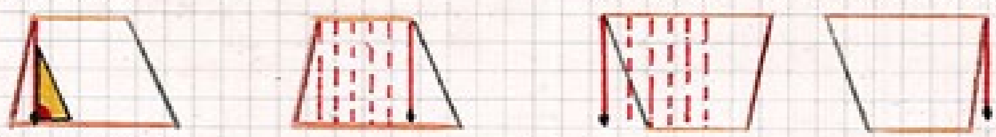
A. This is the quadrilateral region. These are the sides of the common quadrilateral. . .the perimeter, the angles, the vertices of the angles. This figure has no base and therefore has no altitude.

B. The trapezoid. NOTE: The formular used for the area of the trapezoid cannot be applied to the common quadrilateral.

Show the trapezoid in the wooden altitude stand, first on its major base. Using the plumb line, stop at the first vertex, moving right to left along the base. Measure the altitude with the measuring angle. Then move the plumb line to the next vertex, showing the same altitude.

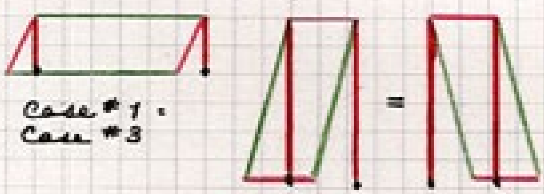
Show the trapezoid on its minor base on the stand, showing the same altitudes.

B. Our nomenclature here is the same: Where is the quadrilateral region? the sides? the vertices? the perimeter? This figure has a base; that is, it has two bases. The bases have to be one of the parallel sides. Neither of the two oblique sides can be the base. If the trapezoid has two bases, it will also have altitudes. This is the first altitude. This is the second altitude. Between the first and the second altitude there are an infinite number of altitudes and they are all the same in length because of the parallel lines.



C. The parallelogram. Examine the altitudes of the parallogram in the stand with the plumb line. NOTE that the parallogram can be turned backwards to show a second position, but it is the same parallelogram and has the same altitudes, etc.

C. Here again we have the same nomenclature for the quadrilateral, but now we have as many bases as there are sides. And thus we have four altitudes. This is the altitude according to this base---and a second according to this base. . .and again we have an infinite number of altitudes between the two because of the parallel lines. Here we show the parallelogram in position #2---again we find an infinite number of altitudes between these two. Then case #3 is the same as case #1. And case #4 equals case #2.



Case #2 =
Case #4

D. The rectangle. Here we have the same nomenclature. And we no longer have the problem of orientation. We repeat the experience of examining the altitudes. We see that each side can be a base.

D. In the rectangle, each of the sides can be a base. And we discover that the altitude is always equal to one of the sides. When the base is one of the longer sides, the altitude is equal to the shorter side. When the base is the shorter side, the altitude is equal to the longer side.

D. Repeat the experience with the rhombus. Here each side can be a base again, and we discover that we have equal and infinite altitudes for each side.

E. In our examination of the square, we discover that the altitude is equal to the base.

GEOMETRIA - GEOMETRY

SOMMA DEGLI ANGOLI INTERNI DI UN POLIGONO

SUM OF THE INTERIOR ANGLES OF A POLYGON

n	......	......	10	9	8	7	6	5	4	3	1. Quanti LATI ha il tuo poligono? 1. How many SIDES has your polygon?
											2. Quante DIAGONALI puoi tracciare da un vertice? 2. How many DIAGONALS can you draw from one vertex?
											3. Quanti TRIANGOLI hai formato? 3. How many TRIANGLES have you formed?
											4. Quanti ANGOLI PIATTI contiene il tuo poligono? 4. How many STRAIGHT ANGLES does your polygon contain?
											5. Come hai ottenuto tale NUMERO di angoli piatti? 5. How did you obtain the NUMBER of straight angles?
											6. A quanti gradi è uguale la SOMMA degli angoli interni del tuo poligono? 6. How many degrees make up the SUM of the interior angles of your polygon?
											7. Quanti gradi misura CIASCUN angolo interno del tuo poligono REGOLARE? 7. How many degrees does EACH interior angle of your REGULAR polygon have?
											8. L'angolo interno del tuo poligono regolare è contenuto ESATTAMENTE nell'angolo giro? (sì/no) 8. Is the interior angle of your regular polygon contained EXACTLY in a whole angle? (yes/no)
											9. PERCHE' con questa piastrella è possibile o perchè non è possibile coprire una superficie? 9. With the corresponding tile WHY is it possible or is it not possible to cover a surface?

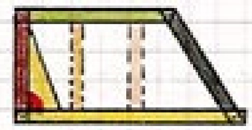
(Final draft of work)

3. Introduce the isosceles trapezoid by constructing it with the sticks, using sticks of equal length for the two oblique sides. Then take the plane figure and demonstrate its equal sides by turning it backwards in the frame. Place the two in display together on the mat, following the scalene.

3. In this trapezoid we have a major and minor base, but the oblique sides that cannot be bases are equal, so it is an isosceles trapezoid.
Here we have used the name of the two oblique sides to give the name to this trapezoid.

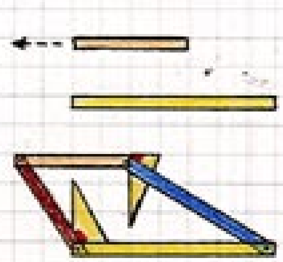
4. Construct the right-angled trapezoid with the sticks:
   a) With two sticks make a right angle, using the measuring angle.
   b) Add a third stick at the top, showing it parallel to the base by measuring with another stick.
   c) Add the fourth stick that will fit between the two parallels.

Display with the corresponding plane figure on the mat.



This is a right-angled trapezoid because it has one of its non-parallel sides perpendicular to its base. Thus we have two right angles.

5. Construct the obtuse -angled trapezoid with the sticks:
   a) Show two sticks, a short and a long. Parallel.
   b) Move the top stick to the left.
   c) With the measuring angle shown resting on the bottom stick, position the third stick at an obtuse angle connecting the parallel sides.
   d) Add the fourth side, showing the measuring angle to indicate another obtuse angle.



Display the constructed obtuse-angled triangle with the plane figure corresponding on the mat. How we show all four together.

This trapezoid must have two obtuse angles.
The first two trapezoids has two obtuse angles, but they were near each other, adjacent.
Here we must have two obtuse angles opposite each other.
Two opposite obtuse angles make this trapezoid an obtuse-angled trapezoid.

6. The child matches the reading labels to the figures.

7. ACTIVITY: The child draws the trapezoids and names them in his notebook.

8. GRAMMAR EXERCISE: Examining the describing phrases here for each of the trapezoids, we see that the name is always the same, but the quality changes. So we can use the adjective grammar box to point this out in the usual exercise.

ON THE REGULARITY AND IRREGULARITY OF POLYGONS

Material

1. The plane insets of the geometry cabinet and the supplementary box of figures and frames.
2. Reading labels: triangle, square, all polygons.

Presentation

1. Invite the child to bring the first drawer (demonstration tray) from the geometry cabinet and to give the name of each figure.

   1. Now this is not just a triangle, but it is an equilateral triangle; a square; a circle.

2. Show the child how to rotate each figure, first the triangle, then the square and the circle, making only one move to the left or right and fitting it again into the frame.

   When I rotate this figure to the right, I must make only one move and then fit it again into the frame. Notice where the angle of the triangle goes as I rotate it. I find that I can also rotate the square and it fits again into the frame. I don't know how much to turn the circle so I put it back.

3. Try the same experience with all the figures in the cabinet. In the drawer of the triangles, only the equilateral triangle, which we have already tried, can be rotated to fit the frame. In the drawer #3, the quadrilaterals, only the square. In the drawer of the polygons, we find that we are able to rotate each one and find the exact fit with one move. The circles drawer presents the indefinite situation. From the sixth drawer of various other figures, we try the two polygons, the trapezoid and the rhombus and discover that neither will work. Finally we attempt the rotation with the supplementary figures and their corresponding card-frames and find no figures which will rotate.

4. Ask the child to now take all those figures from the drawers which could be replaced in the frame by rotating once. He takes the equilateral triangle, the square, and all the polygons and displays them on the mat. (We do not take the circle because we know it is the limit of the polygons and determined by a curved line.)

5. The child matches the reading labels on the mat with the corresponding figures.

6. Repeat the rotation experience in the frames now with each figure, noting that the rotation means that the sides and the angles are equal. Also verbalize this time during the rotation when the figure is outside the frame and when it is inside.

   6. From the seven triangles of reality, we have only 1 triangle; from the six quadrilaterals, we have only the square. And we have the polygons from the pentagon to the decagon. Let's try each figure again in its frame and see how it is able to rotate. If we can turn the figure once it means that all the sides and all the angles are equal. This time tell me when the figure is outside. . .outside. . .in the frame.

7. ACTIVITY (this activity comes after the reading labels, #5): The child draws in his notebook all the displayed figures and writes their names. OR he draws the figures using the frame, then colors them and pastes them in his notebook and writes the name of each.

8. Now, after the #6 exercise, the child takes the paper on which he has shown or drawn the figures. He colors each of the equal angles of the figures red, and traces each of the equal lines in blue. (The sensorial experience of expressing the figures as equiangular and equilateral.

9. CONCLUSION: All the figures which were rotated and fit again into the frames are REGULAR POLYGONS; those that did not fit are IRREGULAR POLYGONS. The child titles his page: My Regular Polygons.

REGULAR AND IRREGULAR POLYGONS. . .

FROM THE IRREGULAR TO THE REGULAR POLYGONS: Second Level

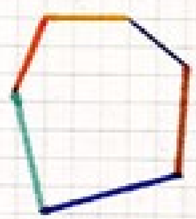
Material

1. The plane figures.
2. The plane, the box of sticks.
3. The cardboard strips which can be cut to any length when needed for a side.
4. A box entitled "Regular and Irregular Polygons" containing:
    a) Two special measuring angles: 108° and 120°.
    b) Three series of reading labels: (these should not be used in the actual presentation when the child should write the labels with the help of the teacher. However, they may be used later when he works with the material alone)
        (1) Series #1: a non-equilateral polygon
        an equilateral polygon
        a non-equiangular polygon
        an equiangular polygon
        (2) Series #2: This is a non-equilateral and non-equiangular polygon.
        This is an equiangular polygon but not an equilateral polygon
        This is an equilateral polygon but not an equiangular polgn.
        This is an equiangular as well as an equilateral polygon.
        (3) Series #3: Irregular Polygon
        Regular Polygon

Presentation

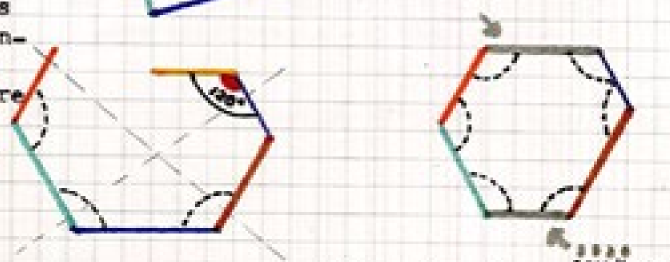
1. Ask the child to take six different sticks and unite them in random order.

1. What can we say about this polygon? It has six sides. It is a hexagon. It does not have equal sides. And we see that it does not have equal angles.



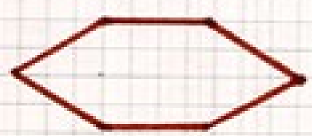
#1 A Hexagon

2. Ask the child to construct another hexagon with the same six sticks, but to unite them using the 120° measuring angle and to show 120° angle between each of the sticks, thus constructing an equiangular hexagon. He finds that he must cut one or two cardboard strips in order to form such a hexagon, eliminating some of the original sticks. He measures each of the angles to make sure that each is 120°.
    NOTE: The hexagon has three diagonals which form 4 triangles.
    Since the three angles of the triangle = 180°,
    the sum of the interior angles of the hexagon = $180^\circ \times 4$.
    $= 720^\circ$
    In the regular hexagon, each angle will be 720° 6, so
    each angle = 120°.



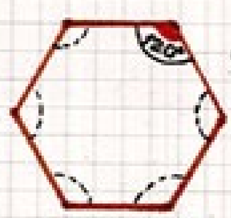
#2 This is an Equiangular Hexagon.

3. Ask the child to construct a hexagon with six sticks of equal length. Show this hexagon with unequal angles on the mat.



#3 An Equilateral Hexagon

4. Ask the child to construct the same hexagon again, but this time to measure with the 120° measuring angle so that all the angles are equal, at 120°.



This is both an Equilateral and an Equiangular Hexagon.