RELATIVE NUMBERS

Age: 10-12

Material

1. The materials of the negative snake game: box of golden 10-bars
   box of bead bars, 1 - 9
   box of grey negative bars, 1 - 9
   box of black & white bead stair
2. Additional bead boxes: box of negative grey 10-bars
   box of yellow & orange stair (for negative changes; comparative to black & white)
3. Two boxes of relative numbers: +1 - +9 in white; -1 - -9 in grey.
4. A box of parentheses, small plus and minus signs.
5.

Now the snake is no longer given to memorize subtraction. In the first exercise we have introduced only the negative bead bars (1 - 9) in grey. The child learned to make addition with two positive bars; and subtraction when one of the bars was grey. We have not mentioned negative numbers.

At this point we must introduce him to algebra as we did with the binomials and trinomials.

Our work with the snake game is now to take to the child's consciousness the concept that it is possible to add negative and positive quantities. So we have addition and subtraction together. The results of the operations may be negative, positive, or zero.

Presentation #1: The Concept of a Negative Result in the Snake Game: Introducing the Materials

1. Introduce the materials and their functions in the snake game. (Grey ten-bars and yellow-orange negative changes are new; show first those snake game materials which the child knows and then present the new.)

1. In the snake game, when we have a result formed of grey bead bars, we cannot take a golden bead bar or a black & white bar.
We take, instead of the black & white a bar from the orange and red stair; and when the result is larger than ten, we use a grey ten-bar.

2. The teacher prepares the first snakes in this work; and should be carefully planned. IN THIS FIRST OPERATION, THE RESULT IS (+6). The snake should begin with one positive bar, then several grey bars, a few positive, a negative. . . A LONG ONE.

3. The child does the work of the snake game as an addition game (or subtraction), bringing down two bars at a time. He verbalizes the operation, then shows the result:
   a) Result of (+10): golden ten in the snake.
   b) Result of more than (+10): ten in the snake and B & W bar as the first term of the next operation.
   c) Result of (-10): grey ten in the snake.
   d) Result of (-10) + : grey ten in the snake and Y & O as first term of the next operation.
   e) Result of less than 10: negative gives Y & O bar as first term of next operation; positive gives B & W as first term of next operation.

3. 9 and 7---9 is a colored bar.
The difference between 9 and 7 is 2; and since 9 (the larger) is a colored bar, I take the 2 B & W bar.
8 and 5---8 is a grey bar.
The difference is 3; 8 is the largest bar and grey, so I take the 3 Y & O bar.
3 and 9---both are grey bars.
3 and 9 are both grey bars; together they give 12, so I put a grey ten-bar in the snake and take the 2 Y & O bar.

NOTE: This simple first verbalization is important. There is no nomenclature yet of positive and negative quantities.
It is the SENSORIAL EXPERIENCE.

NOTE: The child discards the added terms in the box of the B & W stair, now empty so that we can again build the same snake.

4. When the operations are complete, we have a snake formed of golden and grey ten-bars. AND ONE ADDITIONAL BAR, which is generally our remainder, a Black and White (+6).
The child now shows the golden ten-bars vertically positioned on the mat, and matched side by side with the grey tens. They cancel each other---each match giving zero. . . and can be eliminated.

THE RESULT IS (+6).

Presentation #2: A Snake With the Result of Zero

Take the same snake, the bars of which are retained in an empty box as they have been discarded in the first work, and CONSTRUCT A SECOND SNAKE, ADDING A NEGATIVE GREY 6-bar.

The child repeats the work: when he eliminates the transformed tens of the snake, he eliminates all the bars. THE RESULT OF THE SNAKE IS ZERO.

Presentation #3: The Concept of a Negative Number

1. CONSTRUCT A THIRD SNAKE, adding this time a grey (-3) bar. The result of the snake game, then, will be one negative bar (Y & O).

2. Beginning from the observation of this result (-3); give the concept of a negative number.

2. This is a negative number.
We have worked with whole numbers, with decimals and with fractions.
They have always expressed something positive, something absolute.
If I say that I have bought 6 pounds of apples, we know exactly, without a doubt, what quantity I am talking about.
But, when I say that it was very cold this morning---that it was 5°, what do I mean? It is not so clear.

a) Use a centigrade thermometer to show clearly the position of some numbers above and some below zero.

+6
+5
+4
+3
+2
+1
0
-1
-2
-3
-4
-5
-6

-7 -6 -5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5 +6 +7

a) We the thermometer we see two 5s. One is above and one is below zero.
What does the zero represent?
It is the point on the centigrade thermometer at which ice freezes. (water)
It represents coldness, but we know that is it NOT THE COLDEST COLD.
So, if this 2 is above zero, we say 5° C. above zero; and if this 2 is below zero, we say 5° below zero.
WE CAN SHOW THIS ON A DRAWING. . .LIKE THE THERMOMETER. . .
or we can express the SAME THING on a line, which is very much like the way we express B.C. and A.D. onthe time lines.
On the time line, one date such as 30, can be before or after zero (0).

b) Define relative number; and introduce the use of signs.

b) These numbers do no represent something absolute, but something in relation to a certain point. We know what relationship that is because these numbers have a sign in front of them. So they are called RELATIVE NUMBERS.
The numbers with which we have always worked before were absolute numbers:
now we have negative numbers (those preceeded by a minus sign); and positive numbers (those preceeded by a plus sign.)
The numbers are still 1 - 10; but now we use them with either a plus or minus sign; and their value depends on the sign.

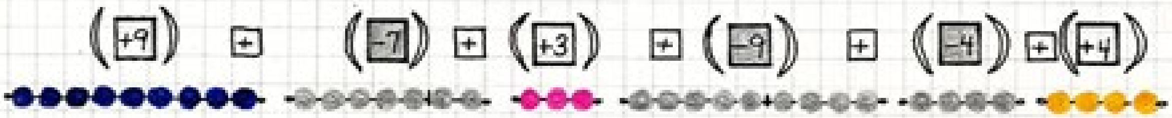
3. Systematically identify all the materials in the boxes as either positive number quantities or negative quantities.

From this point on, we make a fairly rapid progression towards work which eliminates the materials and utilizes ONLY THE NUMERAL CARDS ANDSIGNS.

Presentation #4: Eliminating Like Quantities and Symbols Before the Operation

1. Build a snake, on the pattern of those in the first three presentations. Ask the child to identify each quantity (bar) with the corresponding symbols (using white numeral cards for positive quantities and grey for negative). Then he adds parentheses for each numeral card and shows plus signs between each.

1. All the relative number symbols must be between parentheses; and they must be all added together. . .SO we show addition signs between each term.
This is an algebraic addition.
We are adding positive and negative numbers. So my result can be a positive number, a negative number or zero, according to the greater quantity.



2. Invite the child to look for "like" bars and symbols; he brings those down below the snake on the mat, side by side.
(+9)+(-9) & (+4)+(-4) = 0

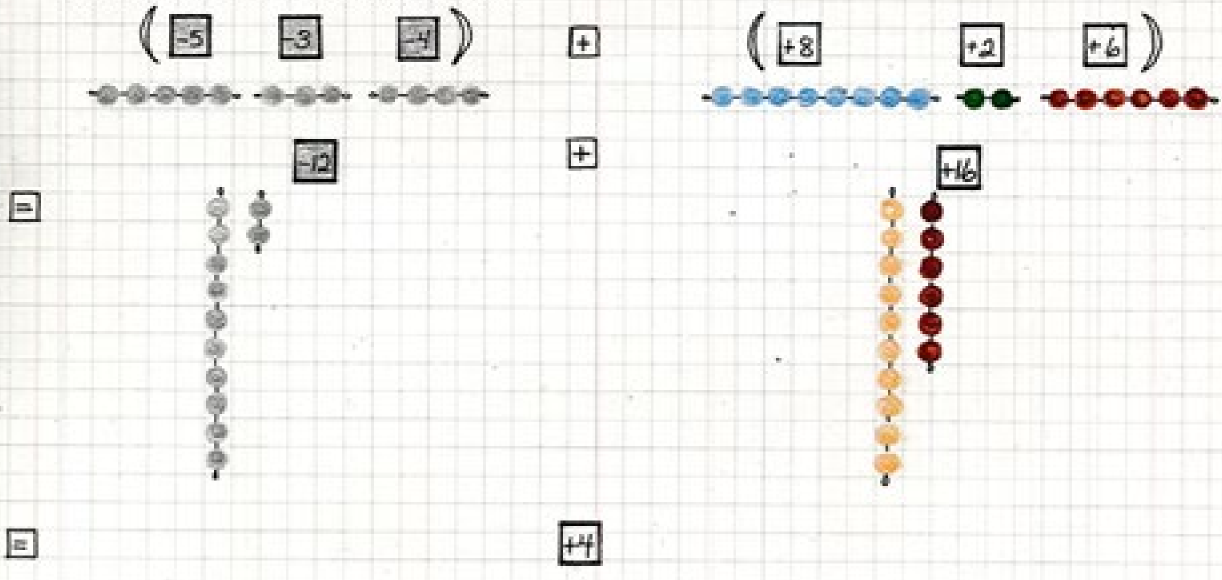
2. To do our operation faster and more efficiently, we can eliminate the positive and negative numbers of the same digit, "like" digits.
We have already done this in the elimination of the grey tens with the golden tens.

3. Complete the snake's addition work and elimination as in previous work.

3. Now the snake we must add is much smaller.

Presentation #5: Collecting the terms

In this snake game, we first eliminate like terms, as above; and then the child puts together all those positive symbols and quantities remaining; and all those negative symbols and quantities before proceeding. He is "collecting the terms," showing the combined terms within one set of parentheses. From this point he shows those collected terms as a sum:



Presentation #6: Written Algebraic Addition

Materials: Without the bars and the symbol cards: the children work in their notebooks.

(+8) + (-9) + (-8) + (-7) + (+7) + (+4) + (+6) +
(-3) + (-6) + (+4) + (+2) =
+8+(-9)+(-8)+(-7)+7+4+6+(-3)+(-6)+4+2 =
+8-9-8-7+7+4+6-3-6+4+2 =

(+4 +4 +2) + (-9 -3) =
+10 + (-12) = -2

When we write the work in our notebooks, we can still simplify the addition:

a)* Eliminate the parentheses
b) Eliminate similar terms.
c) Collect terms.

* *first only those of positive nos.*

Relative Numbers. . .

Presentation #6: Written Algebraic Addition. . .

Formulate the rules: Addition

1. In algebraic addition of relative numbers, if there are several terms of the the same digit and of the same sign, all those equal terms may be grouped as a term of multiplication.
2. In an algebraic addition, opposite addents may be eliminated; the negative and positive addents are all added together (grouped or collected) and we give the result the sign of the largest addent of absolute value.
3. Two relative numbers with the same sign are called concordants. Two relative numbers with different signs are called discordants.

The Different Signs of an Algebraic Operation: A Series of Problems

ADDITION

A. John must receive 15¢ from Lewis and 12¢ from Mark. What quantity will John receive?

$$(+15) + (+12) = +27$$

B. John must give Lewis 10¢ and he must give Mark 5¢. When he pays all his debts, how much less will he have?

$$(-10) + (-5) = -15$$

C. John will receive 25¢ from Lewis, but he must give 17¢ away. When he pays his debt, how much will he have left?

$$(+25) + (-17) = +8$$

D. John must give Mark 12¢ and he will receive from Pete 8¢. When he pays his debt, how much less money will he have?

$$(-12) + (+8) = -4$$

Rules:

1. For the elementary operations with relative numbers, it is necessary to put the operation signs between the numbers. And to avoid confusion, we put the relative numbers between parentheses.
2. The sum of two relative numbers with equal signs is the relative number that takes the sign of the addents. $(-3) + (-6) = (-9)$
3. The sum of two relative numbers with different signs is the relative number that takes the sign of the largest addent. $(-8) + (+12) = +4$

SUBTRACTION

A. Mark has 6¢ and he must give John 8¢. How much less will he have when he pays John?

$$(+6) - (+8) = 6 - 8 = -2$$

B. Mark received 8¢, but he must give John 6¢. However, John doesn't want Mark to pay his 6¢ debt. So he is giving him 6¢. So it is as if Mark now has more.

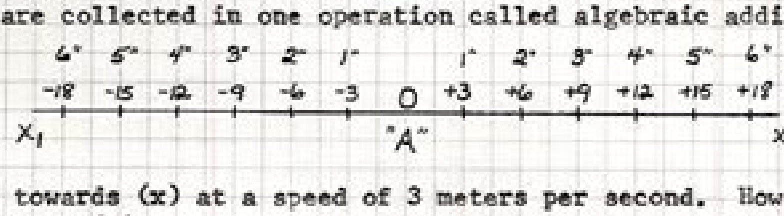
$$(+8) - (-6) = +8 + 6 = +14$$

C. Mark must give John 8¢; but John, seeing that his friend is sad tells him: "I don't want 6¢ of the debt. So because John forgets the debt, of 6¢, he is actually giving him a gift. How much is Mark's debt now.

$$(-8) - (-6) = -8+6 = -2$$

Rule: In reality, addition and subtraction in the relative numbers have no difference; both operations are collected in one operation called algebraic addition.

MULTIPLICATION



A. "A" vehicle travels towards (x) at a speed of 3 meters per second. How many meters does it travel in 4 seconds?

$$(+3) X (+4) = +12$$

B. "A" vehicle travels towards (x_1) at a speed of 3 meters per second. How many meters did it travel after 4 seconds?

$$(+3) X (-4) = -12$$

Relative Numbers. . .
Multiplication Problems. . .

C. From a negative point, A moves towards x, passes through 0 at noon (12 o'clock). Where was "A" 4 seconds before if it travels at the speed of 3 meters per second?

$(-4) \times (+3) = -12$

D. From a positive point, A moves toward (x_1), passing through point 0 at noon. Where was it four seconds before it passed through 0 if it travels at a speed of 3 meters per second? (How many times do I take away 3 meters)

$(-4) \times (-3) = +12$

Rule: The product of two relative numbers corresponds to the product of the single factors. It will have a positive sign if the two factors have the same sign; and a negative sign if the two factors have a different sign.

DIVISION

Material

1. From the materials of memorization of division, the orange box of green beads.
2. A container of silver (grey) beads, loose.
3. Black saucers (made here from napkin rings) marked with a minus sign.
4. Plastic saucers (clear) made here from castors.
5. The box of positive and negative numbers.
6. Box of symbols.
7. 9 little skittles (from the decimal numbers). Others could be used in the three hierarchical colors. Here only unit skittles used.

A. Mary, John and Luke have 12 marbles. They want to divide them among themselves. How many will each one receive.

$(+12) \div (+3) = +4$

(12 green beads in a white saucer)
(3 green unit skittles)
(DISTRIBUTIVE DIVISION: beads distributed)

B. These three children owe 12 marbles to Peter. The three children want to know how many each one of them must give him.

$(-12) \div (+3) = -4$

(12 grey beads in a white saucer)
(3 green unit skittles)
(DISTRIBUTIVE DIVISION: show below each green skittle a white saucer. Distribute the grey beads: the debt)

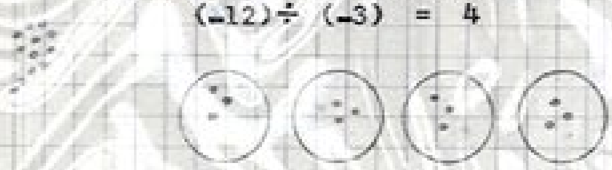
C. The three children want to give Peter a gift of 12 marbles. They want to know how many each of them must contribute.

$(+12) \div (-3) = -4$

(12 green beads shown in a white sa'r)
(3 green skittles representing the children to whom the debt will be distributed AND below each skittle show a black minus saucer---every quantity distributed is an additional unit of debt.)
(DISTRIBUTIVE DIVISION: how many units of debt for each skittle?)

D. Mark has lost, in different games, 12 marbles. He lost three marbles each time. How many games did he lose?

$(-12) \div (-3) = 4$



(12 grey beads---the loss)
(GROUP DIVISION: divide quantity into groups of three, taking an additional white saucer for each group formed.)

Rule: The quotient of the division of two relative numbers is the regular quotient of those two numbers; and it will have a positive sign if they both have the same sign, a negative sign if they have different signs.